Static Electric Field

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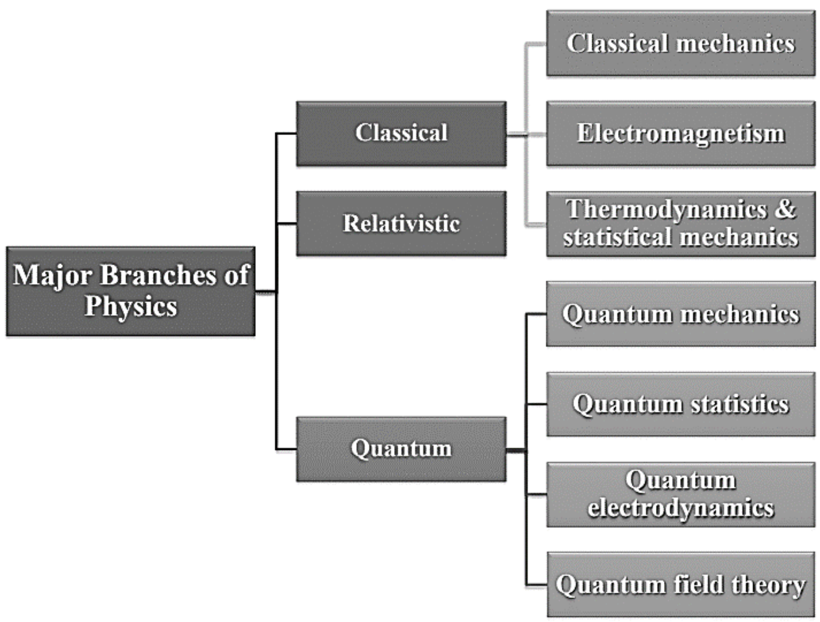
Preliminary lectures

Electricity and Magnetism, Waves and Optics

PHY1121 75 marks [70%(52) Exam, 20% (15)Quizzes, 10% (7.5) Attendance]

Credit: 3, Contact hours/week: 3, Exam time: 3hrs,

Students should answer 6 questions out of 8 taking not more than three in one section.

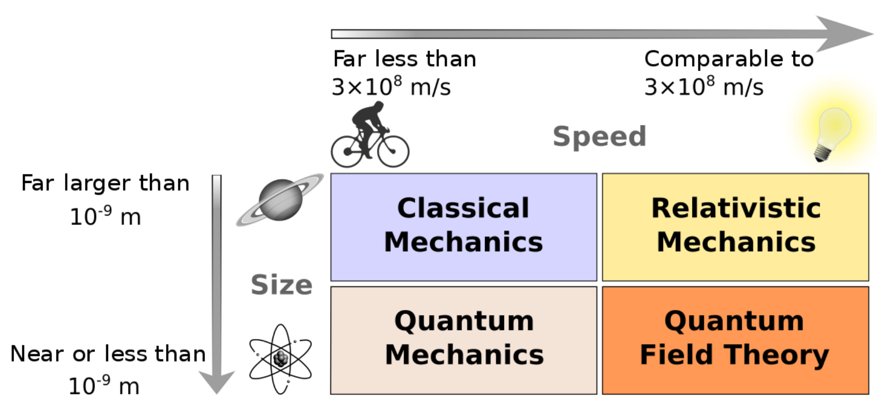


Consideration of size→ **Macroscopic** and **Microscopic**

Macroscopic: Star, planet, etc which visible to naked eye

Microscopic: Atoms, molecules, electron etc which visible with microscope

Road map in physics:



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Section-A:

I. Static Electric Field
II. Static Magnetic Field
III. Electromagnetic Induction
IV. Thermoelectricity

Section-B:

I. Waves
II. Interference of Light
III. Diffraction
IV. Polarization

- Classical physics → Two distinct aspect of nature
    - Matter: localized
    - Energy: Wave, spread in space

Classical Physics

Matter Energy

Modern Physics

Classical physics { Classical mechanics, Electrodynamics } One body system

Modern Physics { Quantum mechanics, Relativity, Statistical mechanics } Many body system

Reference:

Physics part 1 and part 11 – David Halliday and Robert Resnick

Concept of Modern Physics – Arthur Beiser

Electric field

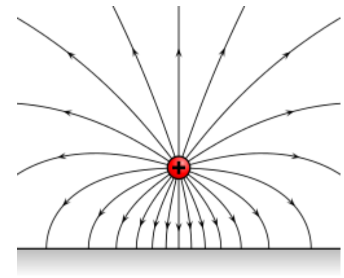

Fig.1: Electric field emanating from a point positive electric charge suspended over an infinite sheet of conducting material.

An **electric field** (sometimes abbreviated as **E-field**) is a vector field surrounding an electric charge that exerts force on other charges, attracting or repelling them. Mathematically the electric field is a vector field that associates to each point in space the force, called the Coulomb force, that would be experienced per unit of charge by an infinitesimal test charge at that point. The units of the electric field in the SI system are newtons per coulomb (N/C), or volts per meter (V/m).
Electric fields are created by electric charges, or by time-varying magnetic fields. On an atomic scale, the electric field is responsible for the attractive force between the atomic nucleus and electrons that holds atoms together, and the forces between atoms that cause chemical bonding. Electric fields and magnetic fields are both manifestations of the electromagnetic force, one of the four fundamental forces (or interactions) of nature.

Electric Lines of Force:

An electric line of force is an imaginary continuous line or curve drawn in an electric field such that tangent to it at any point gives the direction of the electric force at that point. The direction of a line of force is the direction along which a small free positive charge will move along the line. Field lines directed into a closed surface are considered negative; those directed out of a closed surface are positive. If there is no net charge within a closed surface, every field line directed into the surface continues through the interior and is directed outward elsewhere on the surface.

Electric Flux:

Electric flux is a property of an electric field that may be thought of as the number of electric lines of force (or electric field lines) that intersect a given area. Electric field lines are considered to originate on positive electric charges and to terminate on negative charges. If a net charge is contained inside a closed surface, the total flux through the surface is proportional to the enclosed charge, positive if it is positive, negative if it is negative.

The mathematical relation between electric flux and enclosed charge is known as Gauss's law for the electric field, one of the fundamental laws of electromagnetism.

$$\Phi_E = \mathbf{E} \cdot \mathbf{S} = ES\cos\theta.$$

Where **E** is the electric field (having units of V/m), E is its magnitude, S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

For a non-uniform electric field, the electric flux $d\Phi_E$ through a small surface area dS is given by

$$d\Phi_E = \mathbf{E} \cdot d\mathbf{S}$$

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Stokes' theorem: $\oint_{\Gamma} \mathbf{F} \cdot d\mathbf{\Gamma} = \iint_{S} \nabla \times \mathbf{F} \cdot d\mathbf{S}$

Divergence theorem: $\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS$

Gaussian Surface

A Gaussian surface is a closed imaginary surface in three-dimensional space through which the flux of a vector field is calculated. It enclosed all the charges for which flux is to be calculated.

Gauss's Law

The law was first formulated by Joseph-Louis Lagrange in 1773, followed by Carl Friedrich Gauss in 1813. **It is one of Maxwell's four equations**, which form the basis of classical electrodynamics.

The net electric flux through any hypothetical closed surface (Gaussian Surface) is equal to $\frac{1}{\varepsilon_0}$ times the net electric charge within that closed surface.

Integral Form of Gauss's Law

Gauss's law may be expressed as:

$$\Phi_E = \frac{Q}{\varepsilon_0} \qquad (1)$$

Where Φ_E is the electric flux through a closed surface *S* enclosing any volume *V*, *Q* is the total charge enclosed within *V*, and ε_0 is the electric permittivity. The electric flux Φ_E is defined as a surface integral of the electric field:

$$\Phi_E = \oiint_S \mathbf{E} \cdot d\mathbf{A} \qquad (2)$$

Where **E** is the electric field, d**A** is a vector representing an infinitesimal element of area of the surface, and · represents the dot product of two vectors.

Since the flux is defined as an integral of the electric field, this expression of Gauss's law is called the integral form

Differential Form of Gauss's Law

By the divergence theorem, Gauss's law can alternatively be written in the *differential form*:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad (3)$$

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Where $\nabla \cdot \mathbf{E}$ is the divergence of the electric field, ε_0 is the electric constant, and ρ is the total electric charge density (charge per unit volume).

Equivalence of integral and differential forms

The integral and differential forms are mathematically equivalent, by the divergence theorem.

Derivation of Gauss's Law

Let us consider a spherical surface of radius *r* containing an area element ΔA. The solid angle $\Delta\Omega$ subtended at the center of the sphere by this element is defined to be

$$\Delta\Omega \equiv \frac{\Delta A}{r^2} \tag{4}$$

From this equation, we see that $\Delta\Omega$ has no dimensions because ΔA and r^2 both have dimensions L^2. The dimensionless unit of a solid angle is the steradian. Because the surface area of a sphere is $4\pi r^2$, the total solid angle subtended by the sphere is

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ steradians} \tag{5}$$

Now consider a point charge q surrounded by a closed surface of arbitrary shape (Fig. 2). The total electric flux through this surface can be obtained by evaluating for each small area element ΔA and summing over all elements. The flux through each element is

$$\Delta\Phi_E = \mathbf{E} \cdot \Delta\mathbf{A} = E\,\Delta A \cos\theta = k_e q \frac{\Delta A \cos\theta}{r^2} \tag{6}$$

Where r is the distance from the charge to the area element, θ is the angle between the electric field E and ΔA for the element, and $E = \frac{k_e q}{r^2}$ for a point charge. In Fig.3, we see that the projection of the area element perpendicular to the radius vector is $\Delta A cos\theta$. Thus, the quantity $\frac{\Delta A cos\theta}{r^2}$ is equal to the solid angle $\Delta\Omega$, that the surface element ΔA subtends at the charge q. We also see that $\Delta\Omega$ is equal to the solid angle subtended by the area element of a spherical surface of radius r. As the total solid angle at a point is 4π steradians, the total flux through the closed surface is

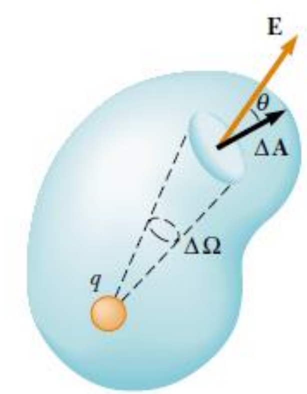

Fig2: A closed surface of arbitrary shape surrounds a point charge *q*. The net electric flux through the surface is independent of the shape of the surface.

$$\Phi_E = k_e q \oint \frac{dA \cos\theta}{r^2} = k_e q \oint d\Omega = 4\pi k_e q = \frac{q}{\epsilon_0} \tag{7}$$

Thus we have derived Gauss's law. Note that this result is independent of the shape of the closed surface and independent of the position of the charge within the surface.

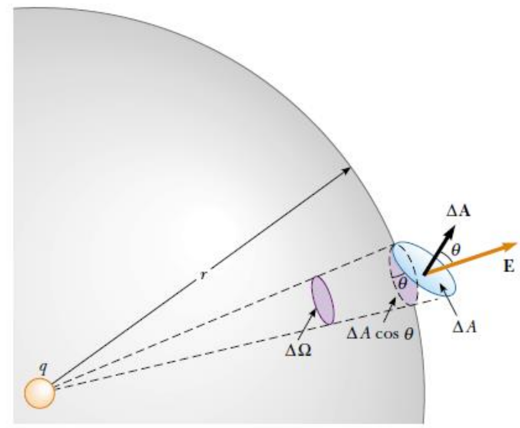

Fig. 3: The area element ΔA subtends a solid angle $\Delta\Omega = \frac{\Delta A cos\theta}{r^2}$ at the charge q.

Application of Gauss's Law to Charged Insulators

In choosing the surface, we should always take advantage of the symmetry of the charge distribution so that we can remove ***E*** from the integral and solve for it. The goal in this type of calculation is to determine a surface that satisfies one or more of the following conditions:

1. The value of the electric field can be argued by symmetry to be constant over the surface.

2. The dot product in ***E.dA*** can be expressed as a simple algebraic product *E dA* because **E** and *d***A** are parallel.

3. The dot product ***E.dA*** is zero when **E** and *d***A** are perpendicular.

4. The field can be argued to be zero over the surface.

The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge *q*.

Solution: A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. We choose a spherical Gaussian surface of radius *r* centered on the point charge, as shown in Fig.4 . The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in condition (2), **E** is parallel to ***d*A** at each point. Therefore, **E.dA**= E dA and Gauss's law gives

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E \, dA = \frac{q}{\epsilon_0} \tag{8}$$

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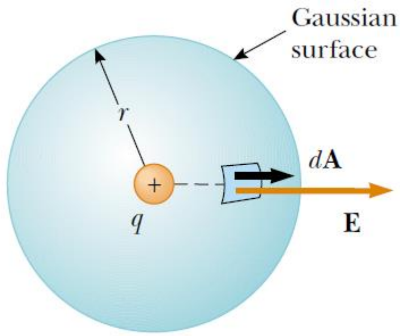

Fig. 4: The point charge q is at the center of the spherical Gaussian surface, and **E** is parallel to **dA** at every point on the surface

By symmetry, *E* is constant everywhere on the surface, which satisfies condition (1), so it can be removed from the integral. Therefore,

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0} \tag{9}$$

Where we have used the fact that the surface area of a sphere is $4\pi r^2$. Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = k_e\frac{q}{r^2} \tag{10}$$

This is the familiar electric field due to a point charge that we found from the Coulomb's law.

A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q. (a) Calculate the magnitude of the electric field at a point outside the sphere. (b) Find the magnitude of the electric field at a point inside the sphere.

Solution (a): As the charge distribution is spherically symmetric, we again select a spherical Gaussian surface of radius r, concentric with the sphere, as shown in Fig.5. For this choice, conditions (1) and (2) are satisfied, as they were for the point charge in previous example, we can write,

$$E = k_e\frac{Q}{r^2} \qquad (\text{for } r > a) \tag{11}$$

Note that this result is identical to the one we obtained for a point charge. Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphere is equivalent to that of a point charge located at the center of the sphere.

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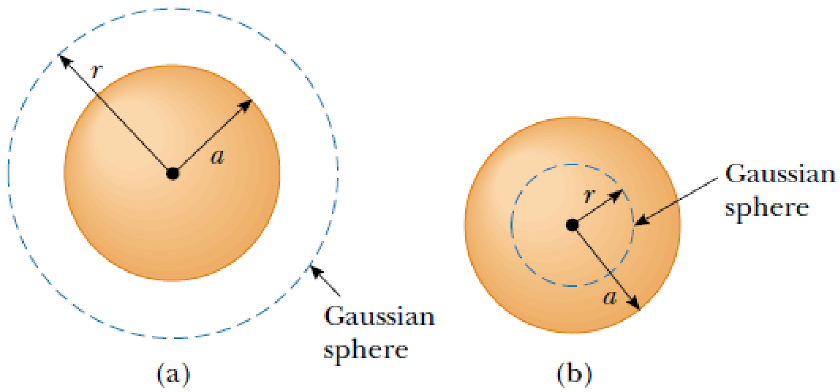

Fig.5: A uniformly charged insulating sphere of radius a and total charge Q.

Solution (b): In this case we select a spherical Gaussian surface having radius r < a, concentric with the insulated sphere (Fig. 5). Let us denote the volume of this smaller sphere by V'. To apply Gauss's law in this situation, it is important to recognize that the charge q_{in} within the Gaussian surface of volume V' is less than Q. To calculate q_{in}, we use the fact that $q_{in} = \rho V'$:

$$q_{in} = \rho V' = \rho \frac{4}{3}\pi r^3 \tag{12}$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical Gaussian surface and is normal to the surface at each point—both conditions (1) and (2) are satisfied. Therefore, Gauss's law in the region r < a gives

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0} \tag{13}$$

Solving for E gives

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r \tag{14}$$

As $\rho = \frac{Q}{\frac{4}{3}\pi r^3}$ by definition and since $k_e = \frac{1}{4\pi r^2}$ this expression for E can be written as

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3} r \qquad (\text{for } r < a) \tag{15}$$

Note that this result for E differs from the one we obtained in part (a). It shows that $E \rightarrow 0$ as $r \rightarrow 0$. Therefore, the result eliminates the problem that would exist at r = 0 if E varied as $1/r^2$ inside the sphere as it does outside the sphere. That is, if $E \propto 1/r2$ for r < a, the field would be infinite at r = 0, which is physically impossible. Note also that the expressions for parts (a) and (b) match when r = a. A plot of E versus r is shown in Fig.6.

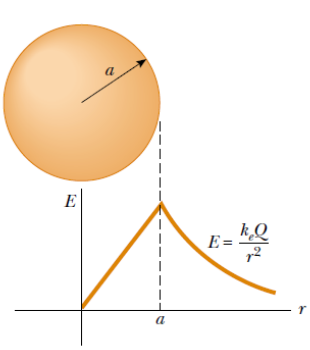

Fig. 6: A plot of E versus r for a uniformly charged insulating sphere. The electric field inside the sphere (r<a) varies linearly with r. The field outside the sphere (r>a) is the same as that of a point charge Q located at r = 0.

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The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig.7). Find the electric field at points (a) outside and (b) inside the shell.

Solution (a): The calculation for the field outside the shell is identical to that for the solid sphere shown in Example 2a. If we construct a spherical Gaussian surface of radius r > a concentric with the shell (Fig. 7b), the charge inside this surface is Q. Therefore, the field at a point outside the shell is equivalent to that due to a point charge Q located at the center:

$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a) \tag{16}$$

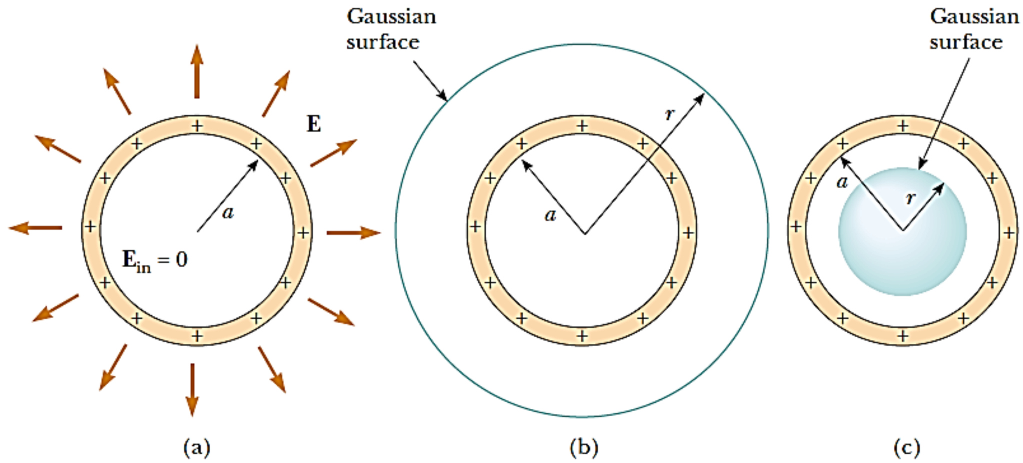

Fig. 7: A thin spherical shell of radius a and charge Q.

Solution (b): The electric field inside the spherical shell is zero. This follows from Gauss's law applied to a spherical surface of radius r < a concentric with the shell (Fig. 7c). Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero—satisfaction of conditions (1) and (2) again—application of Gauss's law shows that E = 0 in the region r < a. We obtain the same results using coulombs law. This calculation is rather complicated. Gauss's law allows us to determine these results in a much simpler way.

A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ

Solution: The symmetry of the charge distribution requires that **E** be perpendicular to the line charge and directed outward, as shown in Fig.8a and b. To reflect the symmetry of the charge distribution, we select a cylindrical Gaussian surface of radius r and length l that is coaxial with the line charge. For the curved part of this surface, **E** is constant in magnitude and perpendicular to the surface at each point—satisfaction of conditions (1) and (2). Furthermore, the flux through the ends of the Gaussian cylinder is zero because **E** is parallel to these surfaces—the first application we have seen of condition (3).

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We take the surface integral in Gauss's law over the entire Gaussian surface. Because of the zero value of **E.**d**A** for the ends of the cylinder, however, we can restrict our attention to only the curved surface of the cylinder. The total charge inside our Gaussian surface is λl. Applying Gauss's law and conditions (1) and (2), we find that for the curved surface

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0} \tag{17}$$

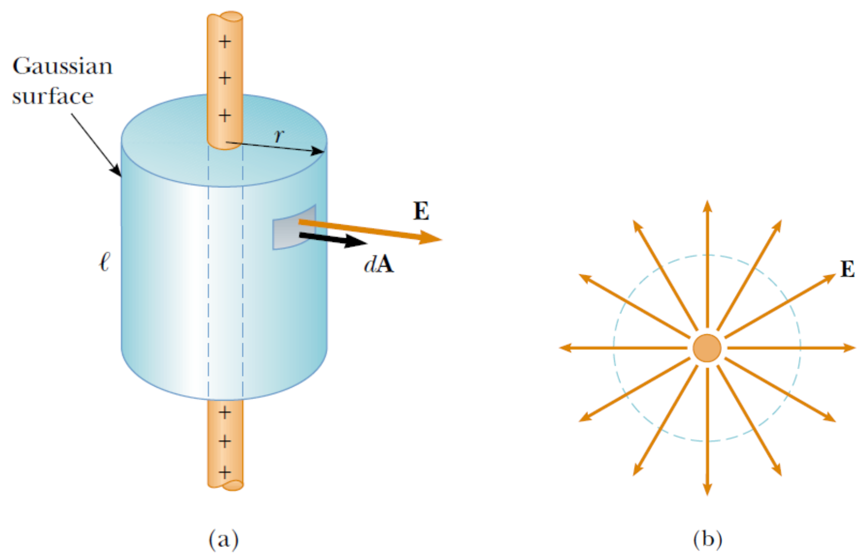

Fig. 8: (a) An infinite line of charge surrounded by a cylindrical Gaussian surface concentric with the line. (b) An end view shows that the electric field at the cylindrical surface is constant in magnitude and perpendicular to the surface.

The area of the curved surface is A=$2\pi r l$ therefore,

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0} \tag{18}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e \frac{\lambda}{r} \tag{19}$$

Thus, we see that the electric field due to a cylindrically symmetric charge distribution varies as 1/r, whereas the field external to a spherically symmetric charge distribution varies as $1/r^2$. If the line charge in this example were of finite length, the result for E would not be that given by the above equation. A finite line charge does not possess sufficient symmetry for us to make use of Gauss's law. This is because the magnitude of the electric field is no longer constant over the surface of the Gaussian cylinder—the field near the ends of the line would be different from that far from the ends. Thus, condition (1) would not be satisfied in this situation. Furthermore, E is not perpendicular to the cylindrical surface at all points—the field vectors near the ends would have a component parallel to the line. Thus, condition (2) would not be satisfied.

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A Nonconducting Plane of Charge

Find the electric field due to a nonconducting, infinite plane of positive charge with uniform surface charge density σ.

Solution:

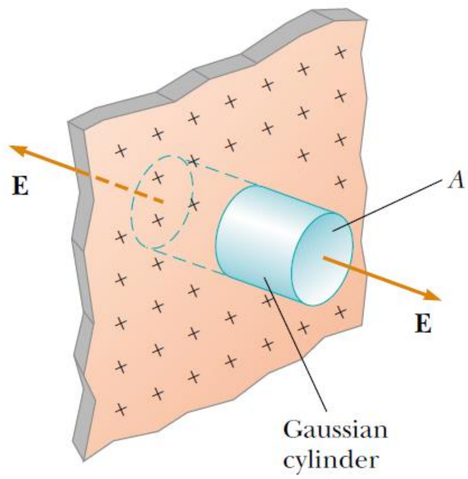

Fig. 9: A cylindrical gaussian surface penetrating an infinite plane of charge

By symmetry, **E** must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane. The fact that the direction of **E** is away from positive charges indicates that the direction of **E** on one side of the plane must be opposite its direction on the other side, as shown in Fig. 9. A Gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane. Because E is parallel to the curved surface—and, therefore, perpendicular to d**A** everywhere on the surface—condition (3) is satisfied and there is no contribution to the surface integral from this surface. For the flat ends of the cylinder, conditions (1) and (2) are satisfied. The flux through each end of the cylinder is EA; hence, the total flux through the entire Gaussian surface is just that through the ends, $\Phi_E = 2EA$.

Noting that the total charge inside the surface is $q_{in} = \sigma A$, we use Gauss's law and find that

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \tag{20}$$

$$E = \frac{\sigma}{2\epsilon_0} \tag{21}$$

Because the distance from each flat end of the cylinder to the plane does not appear in the above equation, we conclude that $E = \frac{\sigma}{2\varepsilon_0}$ at any distance from the plane. That is, the field is uniform everywhere. An important charge configuration related to this example consists of two parallel planes, one positively charged and the other negatively charged, and each with a surface charge density σ. In this situation, the electric fields due to the two planes add in the region between the planes, resulting in a field of magnitude $\frac{\sigma}{\varepsilon_0}$, and cancel elsewhere to give a field of zero.

A Sphere Inside a Spherical Shell

A solid conducting sphere of radius a carries a net positive charge 2Q. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge -Q. Using Gauss's law, find the electric field in the regions labeled 1,2,3 and 4 in Fig.10 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

Solution:

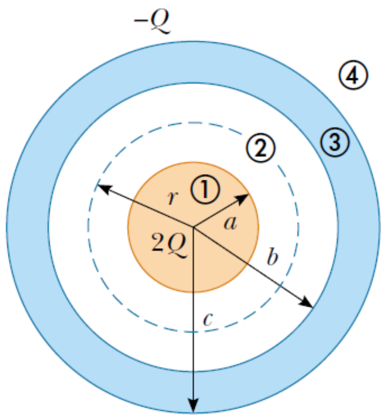

Fig. 10: A solid conducting sphere of radius a and carrying a charge 2Q surrounded by a conducting spherical shell carrying a charge -Q.

First note that the charge distributions on both the sphere and the shell are characterized by spherical symmetry around their common center. To determine the electric field at various distances r from this center, we construct a spherical Gaussian surface for each of the four regions of interest. Such a surface for region 2 is shown in Fig. 10.

To find E inside the solid sphere (region 1), consider a Gaussian surface of radius r < a. Because there can be no charge inside a conductor in electrostatic equilibrium, we see that $q_{in} = 0$; thus, on the basis of Gauss's law and symmetry, $E_1=0$ for r < a.

In region 2—between the surface of the solid sphere and the inner surface of the shell—we construct a spherical Gaussian surface of radius r where a < r < b and note that the charge inside this surface is +2Q (the charge on the solid sphere). Because of the spherical symmetry, the electric field lines must be directed radially outward and be constant in magnitude on the Gaussian surface. Following Example and using Gauss's law, we find that

$$E_2A = E_2(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0} = \frac{2Q}{\epsilon_0} \tag{22}$$

$$E_2 = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{2k_eQ}{r^2} \qquad (\text{for } a < r < b) \tag{23}$$

In region 4, where r > c, the spherical Gaussian surface we construct surrounds a total charge of $q_{in} = +2Q-Q = +Q$. Therefore, application of Gauss's law to this surface gives

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$$E_4 = \frac{k_e Q}{r^2} \qquad (\text{for } r > c)$$

In region 3, the electric field must be zero because the spherical shell is also a conductor in equilibrium. If we construct a Gaussian surface of radius r where b < r < c, we see that q_{in} must be zero because From this argument, we conclude that the charge on the inner surface of the spherical shell must be +2Q to cancel the charge -2Q on the solid sphere. Because the net charge on the shell is -Q, we conclude that its outer surface must carry a charge +Q.

ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

We can calculate the electric potential due to a continuous charge distribution in two ways. If the charge distribution is known, we can calculate the potential for every charges and then sum over the potentials to get the total potential due to the distribution.

Or we can consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 11). The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r} \qquad (24)$$

Where r is the distance from the charge element to point P. To obtain the total potential at point P, we integrate this equation to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point P and because k_e is constant, we can express V as

$$V = k_e \int \frac{dq}{r} \qquad (25)$$

Note that, this expression for V uses a particular reference: The electric potential is taken to be zero when point P is infinitely far from the charge distribution. If the charge distribution is highly symmetric, we first evaluate **E** at any point using Gauss's law and then substitute the value obtained into equation

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (26)$$

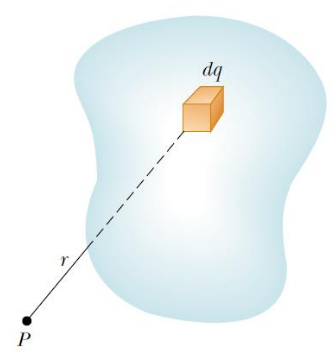

Fig. 11: The electric potential at the point P due to a continuous charge distribution

to determine the potential difference ΔV between any two points. We then choose the electric potential V to be zero at some convenient point.

Electric Potential Due to a Uniformly Charged Ring

(a)Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

(b) Find an expression for the magnitude of the electric field at point P.

Solution (a): Let us orient the ring so that its plane is perpendicular to an x axis and its center is at the origin. We can then take point P to be at a distance x from the center of the ring, as shown in Fig.12. The charge element dq is at a distance $\sqrt{x^2+a^2}$ from point P. Hence, we can express V as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}} \tag{27}$$

Because each element dq is at the same distance from point P we can remove $\sqrt{x^2+a^2}$ from the integral, and V reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}} \tag{28}$$

The only variable in this expression for V is x. This is not surprising because our calculation is valid only for points along the x axis, where y and z are both zero.

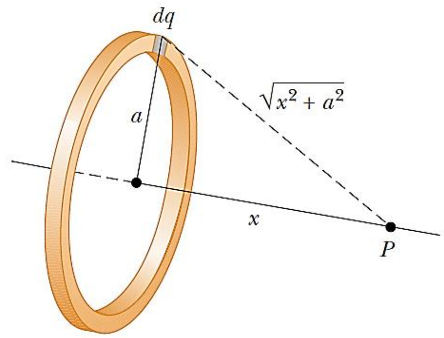

Fig. 12: A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All segments dq of the ring are the same distance from any point P lying on the x axis.

Solution (b): From symmetry, we see that, along the x axis **E** can have only an x component. Therefore, we can write

$$\begin{aligned} E_x &= -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(x^2 + a^2)^{-1/2} \\ &= -k_e Q(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x) \\ &= \boxed{\frac{k_e Q x}{(x^2 + a^2)^{3/2}}} \end{aligned} \tag{29}$$

This result agrees with that obtained by direct integration. Note that $E_x = 0$ at x = 0 (the center of the ring). Could you have guessed this from Coulomb's law?

<u>Exercise:</u> What is the electric potential at the center of the ring? What does the value of the field at the center tell you about the value of V at the center?

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Answer: V = $k_e Q$ /a. Because Ex = - dV/dx = 0 at the center, V has either a maximum or minimum value; it is, in fact, a maximum.

Electric Potential Due to a Uniformly Charged Disk

Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius a and surface charge density σ.

Solution (a): Again, we choose the point P to be at a distance x from the center of the disk and take the plane of the disk to be perpendicular to the x axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 28. Consider one such ring of radius r and width dr, as indicated in Fig.13. The surface area of the ring is dA= 2πrdr ;

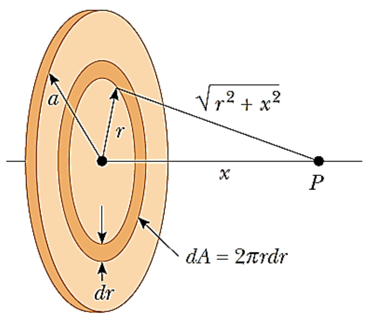

Fig. 13: A uniformly charged disk of radius a lies in a plane perpendicular to the x axis.

From the definition of surface charge density, we know that the charge on the ring is dq = $\sigma dA = \sigma 2\pi$rdr. Hence, the potential at the point P due to this ring is

$$dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}} = \frac{k_e\, \sigma 2\pi r\, dr}{\sqrt{r^2 + x^2}}$$

To find the total electric potential at P, we sum over all rings making up the disk. That is, we integrate dV from r = 0 to r = a:

$$V = \pi k_e \sigma \int_0^a \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2}\, 2r\, dr$$

This integral is of the form $u^n du$ and has the value $un^+1/(n+1)$, where n= -1/2 and u=r^2+x^2. This gives

$$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x] \qquad (30)$$

Solution (b): As in Example 1, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right) \qquad (31)$$

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The calculation of V and **E** for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

Electric Potential Due to a Finite Line of Charge

A rod of length l located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/l$. Find the electric potential at a point P located on the y axis a distance a from the origin.

Solution: The length element dx has a charge dq = λ dx. Because this element is a distance r=$\sqrt{x^2 + a^2}$ from point P, we can express the potential at point P due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at P, we integrate this expression over the limits x = 0 to x = l. Noting that k_e and λ are constants, we find that

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Evaluating V, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right) \tag{32}$$

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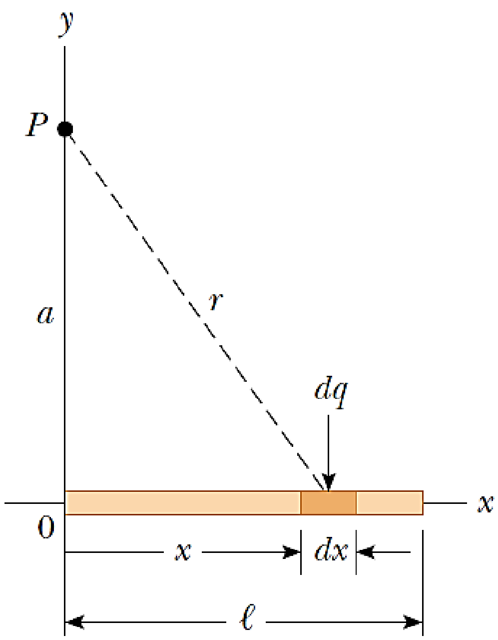

Fig. 14: A uniform line charge of length located along the x axis.

Capacitance and Dielectrics

Consider two conductors carrying charges of equal magnitude but of opposite sign, as shown in Fig. 15. Such a combination of two conductors is called a capacitor. The conductors are called plates. A potential difference ΔV exists between the conductors due to the presence of the charges. Because the unit of potential difference is the volt, a potential difference is often called a voltage. We shall use this term to describe the potential difference across a circuit element or between two points in space.

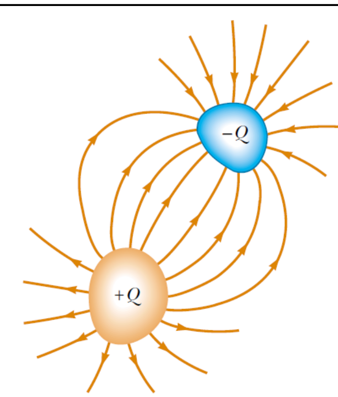

Fig. 15: A capacitor consists of two conductors carrying charges of equal magnitude but opposite sign.

What determines how much charge is on the plates of a capacitor for a given voltage? In other words, what is the *capacity* of the device for storing charge at a particular value of ΔV? Experiments show that the quantity of charge Q on a capacitor is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors. We can write this relationship as $Q = C\Delta V$ if we define capacitance as follows:

The capacitance C of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C = \frac{Q}{\Delta V} \qquad (33)$$

Capacitance has SI units of coulombs per volt. The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday: 1F=1C/V. The farad is a very large unit of capacitance.

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In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F). For practical purposes, capacitors often are labeled "mF" for microfarads and "mmF" for micromicrofarads or, equivalently, "pF" for picofarads.

Parallel-Plate Capacitors

Two parallel metallic plates of equal area A are separated by a distance d, as shown in Fig. 16. One plate carries a charge +Q , and the other carries a charge -Q. Let us consider how the geometry of these conductors influences the capacity of the combination to store charge. Recall that charges of like sign repel one another. As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate. If the capacitor plates are large, the accumulated charges are able to distribute themselves over a substantial area, and the amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Thus, we expect the capacitance to be proportional to the plate area A.

Now let us consider the region that separates the plates. If the battery has a constant potential difference between its terminals, then the electric field between the plates must increase as d is decreased. Let us imagine that we move the plates closer together and consider the situation before any charges have had a chance to move in response to this change. Because no charges have moved, the electric field between the plates has the same value but extends over a shorter distance.

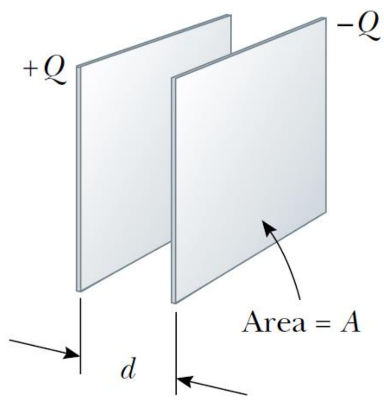

Fig. 15: A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d.

Thus, the magnitude of the potential difference between the plates $\Delta V = Ed$ is now smaller. The difference between this new capacitor voltage and the terminal voltage of the battery now exists as a potential difference across the wires connecting the battery to the capacitor. This potential difference results in an electric field in the wires that drives more charge onto the plates, increasing the potential difference between the plates. When the potential difference between the plates again matches that of the battery, the potential difference across the wires falls back to zero, and the flow of charge stops. Thus, moving the plates closer together causes the charge on the capacitor to

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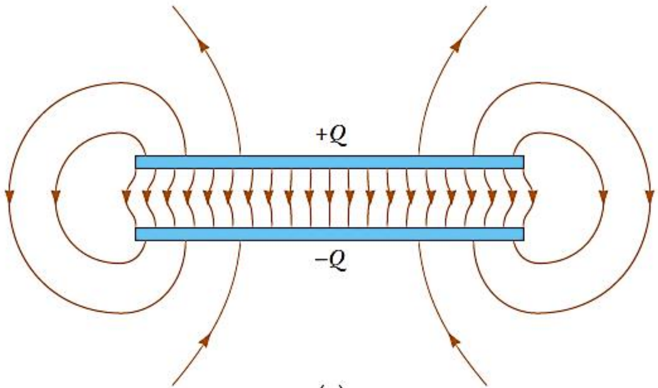
increase. If d is increased, the charge decreases. As a result, we expect the device's capacitance to be inversely proportional to d.



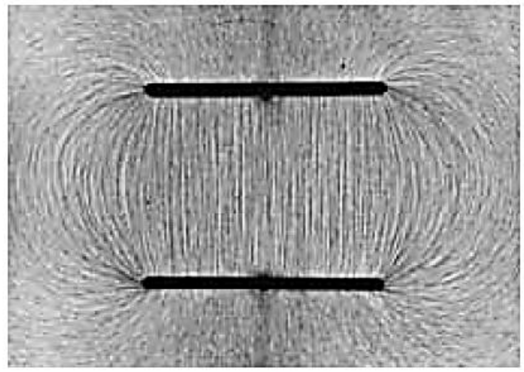


Fig. 17: (a) The electric field between the plates of a parallel-plate capacitor is uniform near the center but nonuniform near the edges. (b) Electric field pattern of two oppositely charged conducting parallel plates. Small pieces of thread on an oil surface align with the electric field.

We can verify these physical arguments with the following derivation. The surface charge density on either plate is $\sigma = \frac{Q}{A}$. If the plates are very close together (in comparison with their length and width), we can assume that the electric field is uniform between the plates and is zero elsewhere. The electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed; therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Substituting this result into Eqn. 33, we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \qquad (34)$$

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That is, **the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation**, just as we expect from our conceptual argument.

The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$ and charge $-Q$. Find the capacitance of this cylindrical capacitor if its length is l.

Solution: It is difficult to apply physical arguments to this configuration, although we can reasonably expect the capacitance to be proportional to the cylinder length l for the same reason that parallel-plate capacitance is proportional to plate area: Stored charges have more room in which to be distributed. If we assume that l is much greater than a and b, we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 18b). We must first calculate the potential difference between the two cylinders, which is given in general by

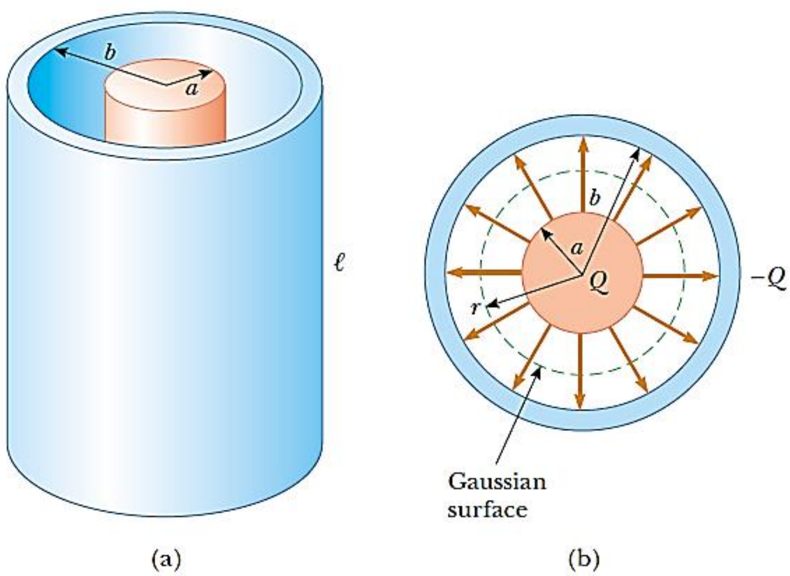

Fig. 18: (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius a and length l surrounded by a coaxial cylindrical shell of radius b. (b) End view. The dashed line represents the end of the cylindrical Gaussian surface of radius r and length l.

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

Where E is the electric field in the region $a < r < b$. We showed using Gauss's law that the magnitude of the electric field of a cylindrical charge distribution having linear charge density λ

is $E_r = 2K_e\frac{\lambda}{r}$. The same result applies here because, according to Gauss's law, the charge on the outer cylinder does not contribute to the electric field inside it. Using this result and noting from Fig.18b that E is along r, we find that

$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda\int_a^b \frac{dr}{r} = -2k_e\lambda\ln\left(\frac{b}{a}\right)$$

Substituting this result into Eqn. 33 and using the fact that $\lambda = \frac{Q}{l}$, we obtain

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{2k_e Q}{\ell}\ln\left(\frac{b}{a}\right)} = \frac{\ell}{2k_e\ln\left(\frac{b}{a}\right)} \qquad (35)$$

Where ΔV is the magnitude of the potential difference, given by $\Delta V = |V_b - V_a| = 2k_e\lambda\,ln(b/a)$, a positive quantity. As predicted, the capacitance is proportional to the length of the cylinders. As we might expect, the capacitance also depends on the radii of the two cylindrical conductors. From Eqn 35, we see that the capacitance per unit length of a combination of concentric cylindrical conductors is

$$\frac{C}{\ell} = \frac{1}{2k_e\ln\left(\frac{b}{a}\right)} \qquad (36)$$

An example of this type of geometric arrangement is a coaxial cable, which consists of two concentric cylindrical conductors separated by an insulator. The cable carries electrical signals in the inner and outer conductors. Such a geometry is especially useful for shielding the signals from any possible external influences.

The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge -*Q* concentric with a smaller conducting sphere of radius a and charge *Q*. Find the capacitance of this device

Solution:

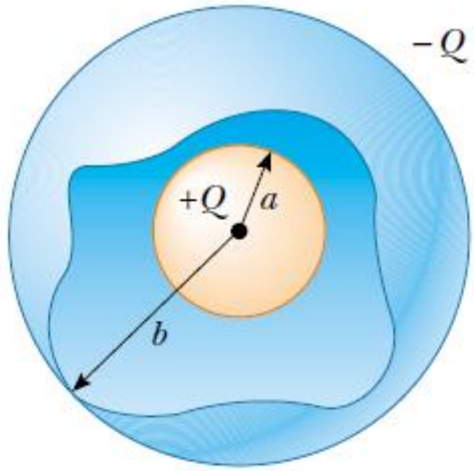

Fig. 19: A spherical capacitor consists of an inner sphere of radius a surrounded by a concentric spherical shell of radius b. The electric field between the spheres is directed radially outward when the inner sphere is positively charged.

As we know that, the field outside a spherically symmetric charge distribution is radial and given by the expression $E=K_eQ/r^2$. In this case, this result applies to the field between the spheres $(a < r < b)$. From Gauss's law we see that only the inner sphere contributes to this field. Thus, the potential difference between the spheres is

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$
$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

The magnitude of the potential difference is

$$\Delta V = |V_b - V_a| = k_eQ\frac{(b-a)}{ab}$$

Substituting this value for ΔV into Eqn 33, we obtain

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)} \qquad (37)$$

Parallel Combination of Capacitors:

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Two capacitors connected as shown in Fig. 20 are known as a parallel combination of capacitors. Figure 26.8b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected by a conducting wire to the positive terminal of the battery and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and are therefore both at the same potential as the negative terminal. Thus, **the individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.** In a circuit such as that shown in Figure 26.8, the voltage applied across the combination is the terminal voltage of the battery. Situations can occur in which the parallel combination is in a circuit with other circuit elements; in such situations, we must determine the potential difference across the combination by analyzing the entire circuit.

When the capacitors are first connected in the circuit shown in Figure 20, electrons are transferred between the wires and the plates; this transfer leaves the left plates positively charged and the right plates negatively charged. The energy source for this charge transfer is the internal chemical energy stored in the battery, which is converted to electric potential energy associated with the charge separation. The flow of charge ceases when the voltage across the capacitors is equal to that across the battery terminals. The capacitors reach their maximum charge when the flow of charge ceases. Let us call the maximum charges on the two capacitors Q_1 and Q_2 . The total charge Q stored by the two capacitors is

$$Q = Q_1 + Q_2 \qquad (38)$$

That is, **the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors**. Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1\,\Delta V \qquad\qquad Q_2 = C_2\,\Delta V$$

Suppose that we wish to replace these two capacitors by one equivalent capacitor having a capacitance Ceq , as shown in Figure 26.8c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store Q units of charge when connected to the battery. We can see from Fig. 20c that the voltage across the equivalent capacitor also is ΔV because the equivalent capacitor is connected directly across the battery terminals. Thus, for the equivalent capacitor,

$$Q = C_{\text{eq}}\,\Delta V$$

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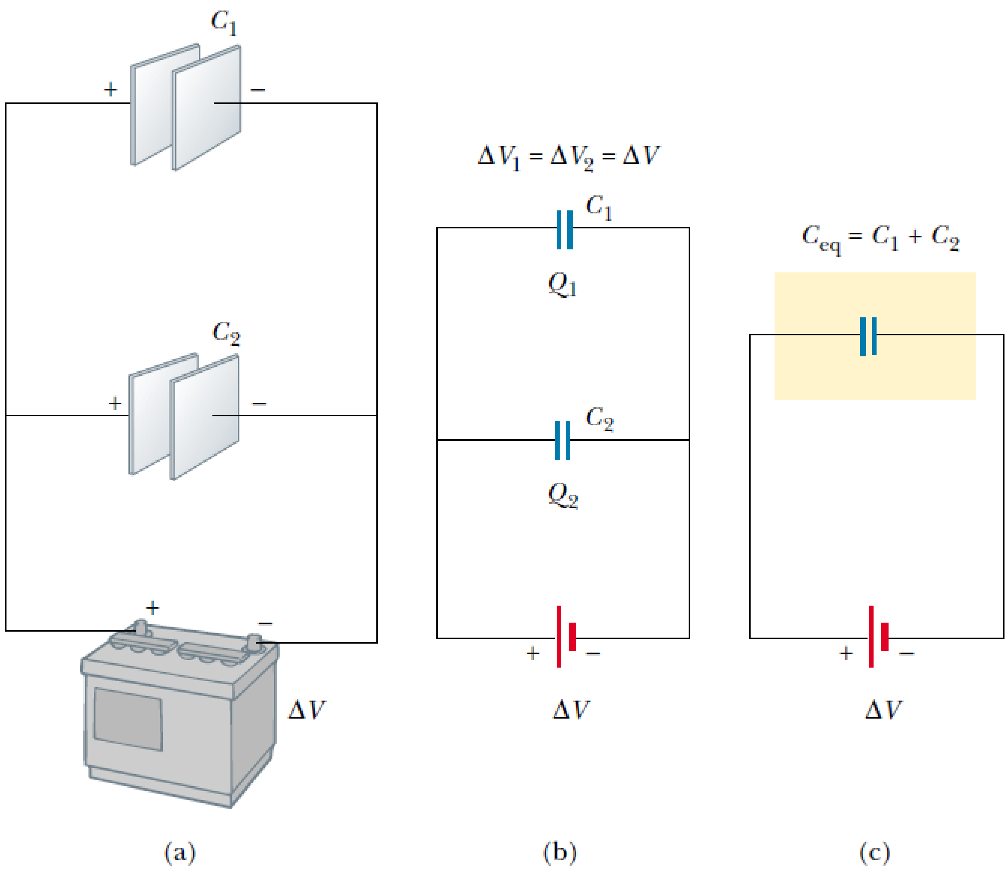

Fig. 20: (a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$

Substituting these three relationships for charge into Eqn. 35, we have

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \left(\begin{array}{l}\text{parallel}\\\text{combination}\end{array}\right)$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)} \tag{39}$$

Thus, **the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.** This makes sense because we are essentially combining the areas of all the capacitor plates when we connect them with conducting wire.

Series Combination

Two capacitors connected as shown in Figure 21a are known as a series combination of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated conductor that is initially uncharged and must continue to have zero net charge. To analyze this combination, let us begin by considering the uncharged capacitors and follow what happens just after a battery is connected to the circuit. When the battery is connected, electrons are transferred out of the left plate of C_1 and into the right plate of C_2. As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is forced off the left plate of C_2, and this left plate therefore has an excess positive charge. The negative charge leaving the left plate of C2 travels through the connecting wire and accumulates on the right plate of C_1. As a result, all the right plates end up with a charge - Q, and all the left plates end up with a charge +Q. Thus, **the charges on capacitors connected in series are the same.**

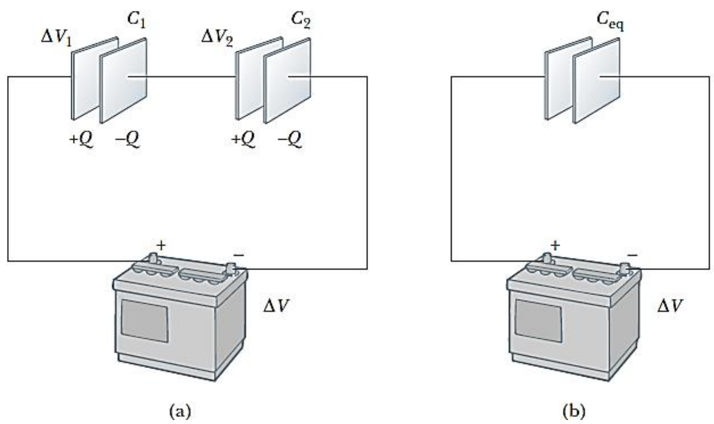

Fig. 21: (a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The capacitors replaced by a single equivalent capacitor.

From Figure 21a, we see that the voltage ΔV across the battery terminals is split between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{40}$$

Where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2, respectively. In general, the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors. Suppose that an equivalent capacitor has the same effect on the circuit as the series combination. After it is fully charged, the equivalent capacitor must have a charge of -Q on its right plate and a charge of +Q on its left plate. Applying the definition of capacitance to the circuit in Figure 21b, we have

$$\Delta V = \frac{Q}{C_{eq}}$$

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Because we can apply the expression $Q=C\Delta V$ to each capacitor shown in Figure 21a, the potential difference across each is

Energy Stored in a Charged Capacitor

Consider a parallel-plate capacitor that is initially uncharged, such that the initial potential difference across the plates is zero. Now imagine that the capacitor is connected to a battery and develops a maximum charge Q. (We assume that the capacitor is charged slowly so that the problem can be considered as an electrostatic system.) When the capacitor is connected to the battery, electrons in the wire just outside the plate connected to the negative terminal move into the plate to give it a negative charge. Electrons in the plate connected to the positive terminal move out of the plate into the wire to give the plate a positive charge. Thus, charges move only a small distance in the wires. To calculate the energy of the capacitor, we shall assume a different process - one that does not actually occur but gives the same final result. We can make this assumption because the energy in the final configuration does not depend on the actual charge-transfer process. We imagine that we reach in and grab a small amount of positive charge on the plate connected to the negative terminal and apply a force that causes this positive charge to move over to the plate connected to the positive terminal. Thus, we do work on the charge as we transfer it from one plate to the other. At first, no work is required to transfer a small amount of charge *dq* from one plate to the other. However, once this charge has been transferred, a small potential difference exists between the plates. Therefore, work must be done to move additional charge through this potential difference. As more and more charge is transferred from one plate to the other, the potential difference increases in proportion, and more work is required.

Suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. We know that the work necessary to transfer an increment of charge *dq* from the plate carrying charge -*q* to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

The total work required to charge the capacitor from $q=0$ to some final charge $q=Q$ is

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

The work done in charging the capacitor appears as electric potential energy *U* stored in the capacitor. Therefore, we can express the potential energy stored in a charged capacitor in the following forms:

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \qquad (41)$$

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This result applies to any capacitor, regardless of its geometry. We see that for a given capacitance, the stored energy increases as the charge increases and as the potential difference increases. In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently great value of ΔV, discharge ultimately occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship ΔV = Ed. Furthermore, its capacitance is $C = \epsilon_0 A/d$. Substituting these expressions into Equation 41, we obtain

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2 \qquad (42)$$

Because the volume V (volume, not voltage!) occupied by the electric field is Ad, the energy per unit volume $U_E = \frac{U}{V} = \frac{U}{Ad}$ known as the energy density, is

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (43)$$

Although Equation 43 was derived for a parallel-plate capacitor, the expression is generally valid. That is, **the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.**

Capacitors with Dielectrics

A **dielectric** is a nonconducting material, such as rubber, glass, or waxed paper. When a dielectric is inserted between the plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor *K*, which is called the **dielectric constant**. The dielectric constant is a property of a material and varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference.

We can perform the following experiment to illustrate the effect of a dielectric in a capacitor: Consider a parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0. The potential difference across the capacitor is $\Delta V_0 = \frac{Q_0}{C_0}$. Figure 22a illustrates this situation. The potential difference is measured by a voltmeter.

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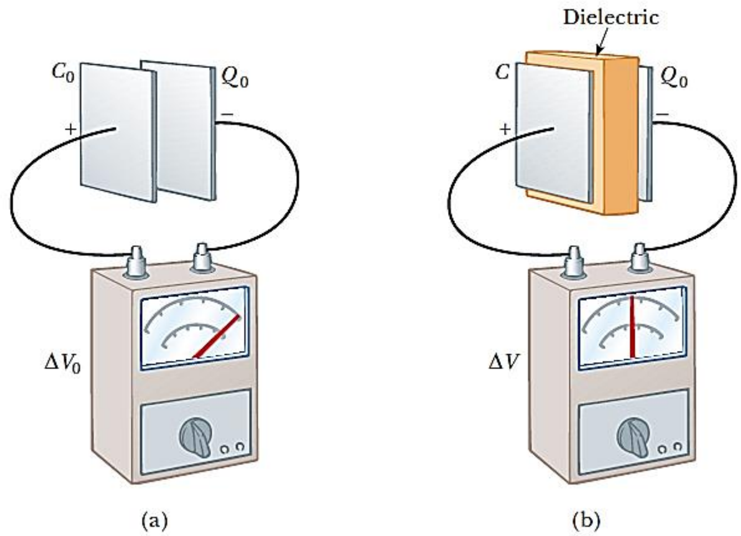

Fig.22: Charged capacitor (a) before and (b) after insertion of a dielectric between the plates.

Note that no battery is shown in the figure; also, we must assume that no charge can flow through an ideal voltmeter. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates, as shown in Figure 22b, the voltmeter indicates that the voltage between the plates decreases to a value ΔV. The voltages with and without the dielectric are related by the factor *K* as follows:

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

Because $\Delta V < \Delta V_0$, we see that k>1. Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

That is, the capacitance inc $C = \kappa C_0$ tric completely fills the region (44) between the plates. For a parallel-plate capacitor, where C=$\frac{\epsilon_0 A}{d}$, we can express the capacitance when the capacitor is filled with a dielectric as

$$C = k\frac{\epsilon_0 A}{d} \qquad (45)$$

From Equation 45, it would appear that we could make the capacitance very large by decreasing d, the distance between the plates. In practice, the lowest value of d is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength (maximum electric field) of the dielectric. If the magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct. Insulating materials have values of k greater than unity and dielectric strengths greater than that of air, as Table 1 indicates. Thus, we see that a dielectric provides the following advantages:

• Increase in capacitance
• Increase in maximum operating voltage
• Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

Table 1: Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature Dielectric

Material	**Dielectric Constant κ**	**Dielectric Strength[a] (V/m)**
Air (dry)	1.000 59	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Polyvinyl chloride	3.4	40×10^6
Porcelain	6	12×10^6
Pyrex glass	5.6	14×10^6
Silicone oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Vacuum	1.000 00	—
Water	80	—

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Electric Dipole in an Electric Field

The electric dipole consists of two charges of equal magnitude but opposite sign separated by a distance 2a, as shown in Figure 23. The electric dipole moment of this configuration is defined as the vector p directed from -q to +q along the line joining the charges and having magnitude 2aq:

$$p \equiv 2aq \quad (46)$$

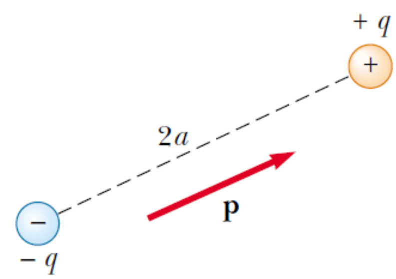

Fig. 23: An electric dipole consists of two charges of equal magnitude but opposite sign separated by a distance of 2a. The electric dipole moment p is directed from - q to +q.

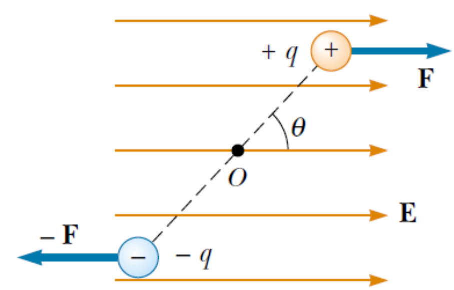

Fig. 24: An electric dipole in a uniform external electric field. The dipole moment p is at an angle θ to the field, causing the dipole to experience a torque.

Now suppose that an electric dipole is placed in a uniform electric field E, as shown in Figure 24. We identify E as the field external to the dipole, distinguishing it from the field due to the dipole. The field E is established by some other charge distribution, and we place the dipole into this field. Let us imagine that the dipole moment makes an angle with the field. The electric forces acting on the two charges are equal in magnitude but opposite in direction as shown in Figure 24 (each has a magnitude$F = qE$. Thus, the net force on the dipole is zero. However, the two forces produce a net torque on the dipole; as a result, the dipole rotates in the direction that brings the dipole moment vector into greater alignment with the field. The torque due to the force on the positive charge about an axis through O in Figure 24 is Fa sinθ , where asinθ is the moment arm of F about O. This force tends to produce a clockwise rotation. The torque about O on the negative charge also is Fa sin ; here again, the force tends to produce a clockwise rotation. Thus, the net torque about O is

$$\tau = 2Fasin\theta$$

Because $F = qE$ and $p = 2aq$ we can express τ as

$$\tau = 2aqEsin\theta$$

$$= pEsin\theta \quad (47)$$

It is convenient to express the torque in vector form as the cross product of the vectors **p** and **E:**

$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E} \quad (48)$$

We can determine the potential energy of the system of an electric dipole in an external electric field as a function of the orientation of the dipole with respect to the field. To do this, we recognize that work must be done by an external agent to rotate the dipole through an angle so as to cause the dipole moment vector to become less aligned with the field. The work done is then stored as potential energy in the system of the dipole and the external field. The work dW required to rotate the dipole through an angle dθ is $dW = \tau d\theta$. Because $\tau = pEsin\theta$ and because the work is transformed into potential energy U, we find that, for a rotation from θ_i to θ_f, the change in potential energy is

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$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta \, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$
$$= pE \Big[-\cos\theta \Big]_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

The term that contains $\cos\theta_i$ is a constant that depends on the initial orientation of the dipole. It is convenient for us to choose $\theta_i = 0$, so that $\cos\theta_i = \cos 90 = 0$. Furthermore, let us choose $U_i = 0$ at $\theta_i = 90^o$ as our reference of potential energy. Hence, we can express a general value of $U = U_f$ as

$$U = -pE\cos\theta \quad (49)$$

We can write this expression for the potential energy of a dipole in an electric field as the dot product of the vectors **p** and **E:**

$$U = -\boldsymbol{p}.\boldsymbol{E} \quad (50)$$

Molecules are said to be polarized when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule. In some molecules, such as water, this condition is always present—such molecules are called polar molecules. Molecules that do not possess a permanent polarization are called nonpolar molecules.

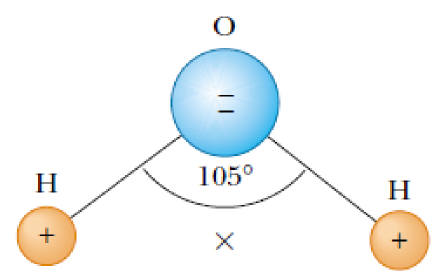

Fig. 25: The water molecule, H2O, has a permanent polarization resulting from its bent geometry. The center of the positive charge distribution is at the point x.

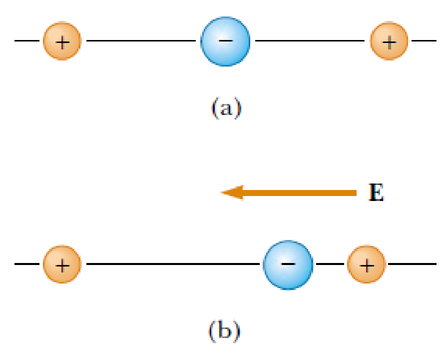

Fig. 26: (a) A symmetric molecule has no permanent polarization. (b) An external electric field induces a polarization in the

We can understand the permanent polarization of water by inspecting the geometry of the water molecule. In the water molecule, the oxygen atom is bonded to the hydrogen atoms such that an angle of 105° is formed between the two bonds (Fig. 25). The center of the negative charge distribution is near the oxygen atom, and the center of the positive charge distribution lies at a point midway along the line joining the hydrogen atoms (the point labeled in Fig. 25). We can model the water molecule and other polar molecules as dipoles because the average positions of the positive and negative charges act as point charges. As a result, we can apply our discussion of dipoles to the behavior of polar molecules.

Microwave ovens take advantage of the polar nature of the water molecule. When in operation, microwave ovens generate a rapidly changing electric field that causes the polar molecules to swing back and forth, absorbing energy from the field in the process. Because the jostling molecules collide with each other, the energy they absorb from the field is converted to internal energy, which corresponds to an increase in temperature of the food.

Another household scenario in which the dipole structure of water is exploited is washing with soap and water. Grease and oil are made up of nonpolar molecules, which are generally not attracted to water. Plain water is not very useful for removing this type of grime. Soap contains long molecules called surfactants. In a long molecule, the polarity characteristics of one end of the molecule can be different from those at the other end. In a surfactant molecule, one end acts like a nonpolar molecule and the other acts like a polar molecule. The nonpolar end can attach to a grease or oil molecule, and the polar end can attach to a water molecule. Thus, the soap serves as a chain, linking the dirt and water molecules together. When the water is rinsed away, the grease and oil go with it.

A symmetric molecule (Fig. 26a) has no permanent polarization, but polarization can be induced by placing the molecule in an electric field. A field directed to the left, as shown in Figure 26b, would cause the center of the positive charge distribution to shift to the left from its initial position and the center of the negative charge distribution to shift to the right. This induced polarization is the effect that predominates in most materials used as dielectrics in capacitors.

An Atomic Description of Dielectrics

We know that the potential difference ΔV_0 between the plates of a capacitor is reduced to $\Delta V_0/k$when a dielectric is introduced. Because the potential difference between the plates equals the product of the electric field and the separation d, the electric field is also reduced. Thus, if $\mathbf{E_0}$ is the electric field without the dielectric, the field in the presence of a dielectric is

$$\mathbf{E}=\frac{\boldsymbol{E_o}}{\boldsymbol{k}} \tag{51}$$

Let us first consider a dielectric made up of polar molecules placed in the electric field between the plates of a capacitor. The dipoles (that is, the polar molecules making up the dielectric) are randomly oriented in the absence of an electric field, as shown in Figure 27a. When an external field $\mathbf{E_0}$ due to charges on the capacitor plates is applied, a torque is exerted on the dipoles, causing them to partially align with the field, as shown in Figure 27b. We can now describe the dielectric as being polarized. The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the field. In general, the alignment increases with decreasing temperature and with increasing electric field.

If the molecules of the dielectric are nonpolar, then the electric field due to the plates produces some charge separation and an *induced dipole moment*. These induced dipole moments tend to align with the external field, and the dielectric is polarized. Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or nonpolar.

With these ideas in mind, consider a slab of dielectric material placed between the plates of a capacitor so that it is in a uniform electric field $\mathbf{E_0}$**,** as shown in Figure 28a. The electric field due to the plates is directed to the right and polarizes the dielectric. The net effect on the dielectric is the formation of an induced positive surface charge density $+\sigma_{\text{ind}}$ on the right face and an equal negative surface charge density - σ_{ind} on the left face, as shown in Figure 26.24b. These induced

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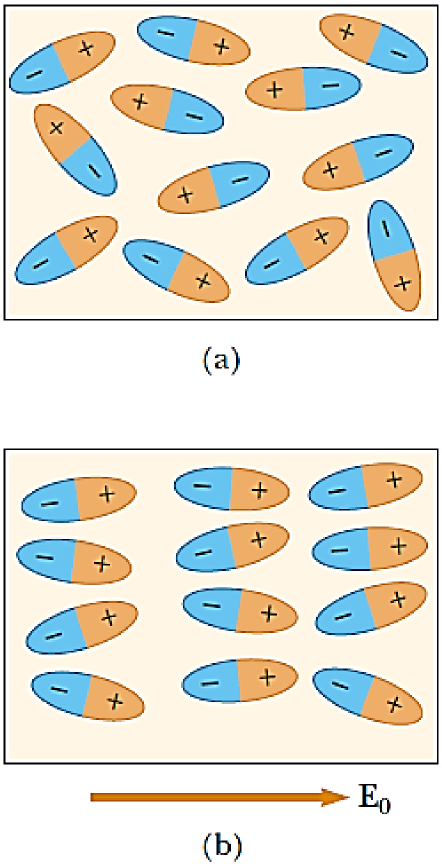

Fig. 27: (a) Polar molecules are randomly oriented in the absence of an external electric field. (b) When an external field is applied, the molecules partially align with the field.

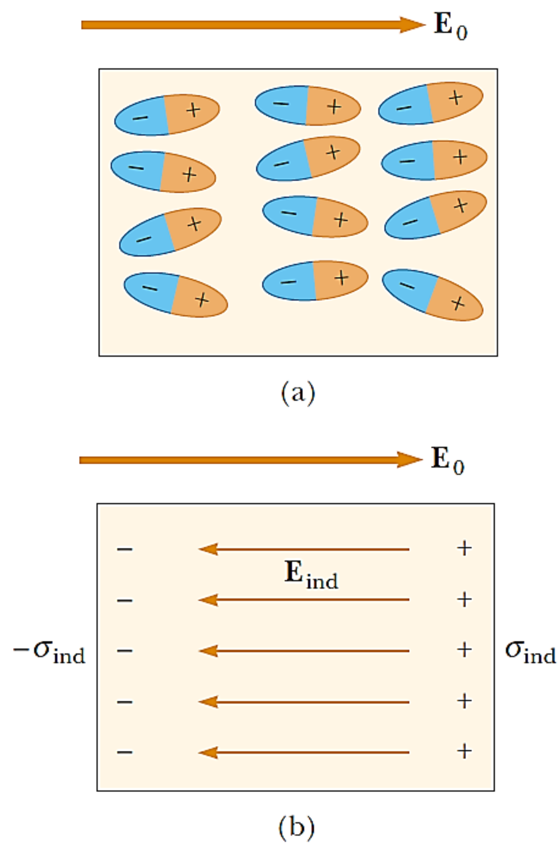

Fig. 28: (a) When a dielectric is polarized, the dipole moments of the molecules in the dielectric are partially aligned with the external field E_0. (b) This polarization causes an induced negative surface charge on one side of the dielectric and an equal induced positive surface charge on the opposite side. This separation of charge results in a reduction in the net electric field within the dielectric.

surface charges on the dielectric give rise to an induced electric field E_{ind} in the direction opposite the external field $\mathbf{E_0}$. Therefore, the net electric field **E** in the dielectric has a magnitude

$$E = E_0 - E_{\text{ind}} \tag{52}$$

In the parallel-plate capacitor shown in Figure 29, the external field $\mathbf{E_0}$ is related to the charge density σ on the plates through the relationship $E = \frac{\sigma}{\epsilon_0}$. The induced electric field in the dielectric is related to the induced charge density σ_{ind} through the relationship $E_{ind} = \frac{\sigma_{ind}}{k}$. Because $E = E_0/k = \sigma / \epsilon_0$ substitution into Equation 52 gives

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$$\frac{\sigma}{\kappa\epsilon_0} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{\text{ind}}}{\epsilon_0}$$

$$\sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma$$

Because k>1 this expression shows that the charge density σ_{ind} induced on the dielectric is less than the charge density σ on the plates. For instance, if k=3 we see that the induced charge density is two-thirds the charge density on the plates. If no dielectric is present, then k=1 and $\sigma_{\text{ind}} = 0$ as expected. However, if the dielectric is replaced by an electrical conductor, for which E=0 then Eqn.52 indicates that this corresponds to $\sigma_{\text{ind}} = \sigma$. That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor.

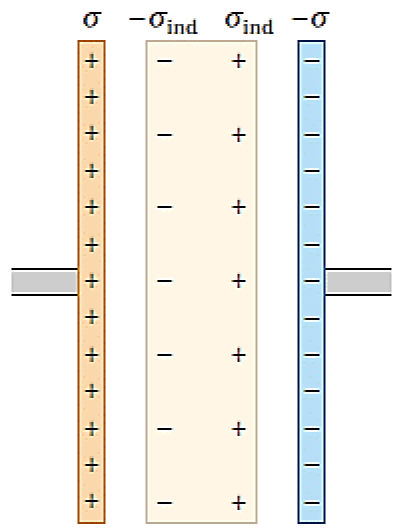

Fig. 29: Induced charge on a dielectric placed between the plates of a charged capacitor. Note that the induced charge density on the dielectric is less than the charge density on the plates.

Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. (a) Find the capacitance of the device.

A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant *k* and thickness d/3 is inserted between the plates?

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