Electromagnetic Induction

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Faraday’s law of electromagnetic induction

To see how an emf can be induced by a changing magnetic field, let us consider a loop of wire connected to a galvanometer, as illustrated in Figure 1. When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right in Figure 1a. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in Figure 1c. When the magnet is held stationary relative to the loop (Fig. 1b), no deflection is observed. Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects. From these observations, we conclude that the loop “knows” that the magnet is moving relative to it because it experiences a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field. These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit! We call such a current an induced current and say that it is produced by an induced emf.

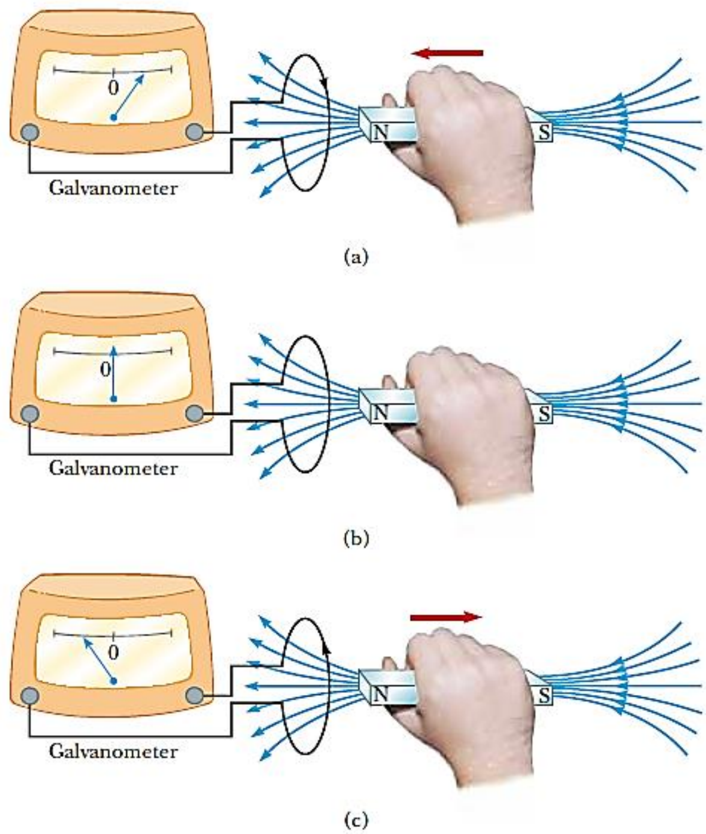

Fig. 1: (a) When a magnet is moved toward a loop of wire connected to a galvanometer, the galvanometer deflects as shown, indicating that a current is induced in the loop. (b) When the magnet is held stationary, there is no induced current in the loop, even when the magnet is inside the loop. (c) When the magnet is moved away from the loop, the galvanometer deflects in the opposite direction, indicating that the induced current is opposite that shown in part (a). Changing the direction of the magnet’s motion changes the direction of the current induced by that motion.

Now let us describe an experiment conducted by Faraday and illustrated in Figure 2. A primary coil is connected to a switch and a battery. The coil is wrapped around a ring, and a current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a galvanometer. No battery is present in the secondary circuit, and the secondary coil is not connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent. Initially, you might guess that no current is ever detected in the secondary circuit. However, something quite amazing happens when the switch in the primary circuit is either suddenly closed or suddenly opened. At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero. Finally, the galvanometer reads zero when there is either a steady current or no current in the primary circuit.

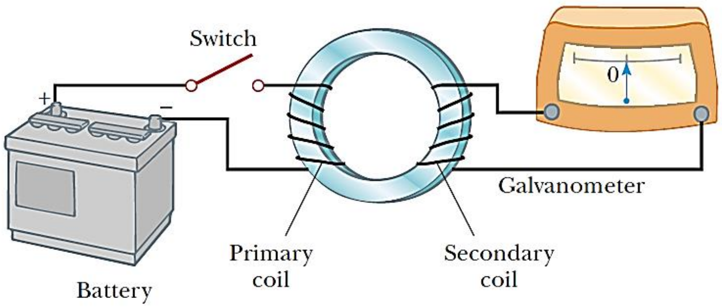

Fig.2: Faraday's experiment. When the switch in the primary circuit is closed, the galvanometer in the secondary circuit deflects momentarily. The emf induced in the secondary circuit is caused by the changing magnetic field through the secondary coil.

The key to understanding what happens in this experiment is to first note that when the switch is closed, the current in the primary circuit produces a magnetic field in the region of the circuit, and it is this magnetic field that penetrates the secondary circuit. Furthermore, when the switch is closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and it is this changing field that induces a current in the secondary circuit.

As a result of these observations, Faraday concluded that an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field. The induced current exists for only a short time while the magnetic field through the secondary coil is changing. Once the magnetic field reaches a steady value, the current in the secondary coil disappears. In effect, the secondary circuit behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the secondary circuit by the changing magnetic field.

The experiments shown in Figures 1 and 2 have one thing in common: In each case, an emf is induced in the circuit when the magnetic flux through the circuit changes with time.

In general**, the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit**. This statement, known as **Faraday's law of induction**, can be written

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt} \quad (1$$

Where $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ is the magnetic flux through the circuit. If the circuit is a coil consisting of N loops all of the same area and if Φ_B is the flux through one loop, an emf is induced in every loop; thus, the total induced emf in the coil is given by the expression

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$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad (2$$

The negative sign in Equations 1 and 2 is of important physical significance, which we shall discuss in Lenz law. Suppose that a loop enclosing an area A lies in a uniform magnetic field B, as shown in Figure 3. The magnetic flux through the loop is equal to $BA \cos\theta$.

Hence, the induced emf can be expressed as

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta) \qquad (3$$

From this expression, we see that an emf can be induced in the circuit in several ways:

• The magnitude of **B** can change with time.
• The area enclosed by the loop can change with time.
• The angle θ between **B** and the normal to the loop can change with time.
• Any combination of the above can occur.

Some Applications of Faraday's Law

The Ground Fault Interrupter (GFI of GFCI)

The ground fault interrupter is an interesting safety device that protects users of electrical appliances against electric shock. Its operation makes use of Faraday's law. In the GFI shown in Figure 3, wire 1 leads from the wall outlet to the appliance to be protected, and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounds the two wires, and a sensing coil is wrapped around part of the ring. Because the currents in the wires are in opposite directions, the net magnetic flux through the sensing coil due to the currents is zero. However, if the return current in wire 2 changes, the net magnetic flux through the sensing coil is no longer zero. (This can happen, for example, if the appliance gets wet, enabling current to leak to ground.) Because household current is alternating (meaning that its direction keeps reversing), the magnetic flux through the sensing coil changes with time, inducing an emf in the coil. This induced emf is used to trigger a circuit breaker, which stops the current before it is able to reach a harmful level.

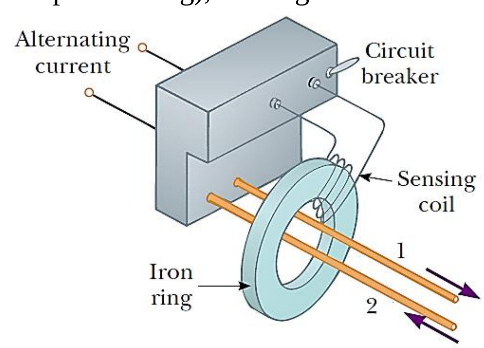

Fig. 3: Essential components of a ground fault interrupter.

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Production of Sound in an Electric Guitar

Another interesting application of Faraday's law is the production of sound in an electric guitar (Fig. 4). The coil in this case, called the pickup coil, is placed near the vibrating guitar string, which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the portion of the string nearest Lenz's law the coil. When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces an emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produce the sound waves we hear.

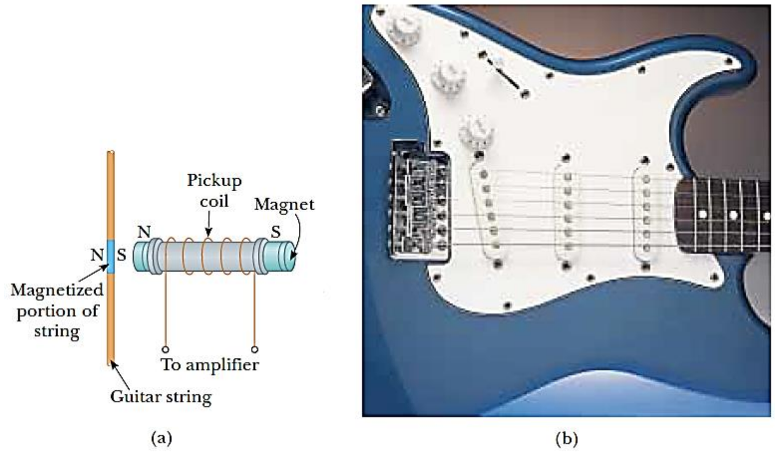
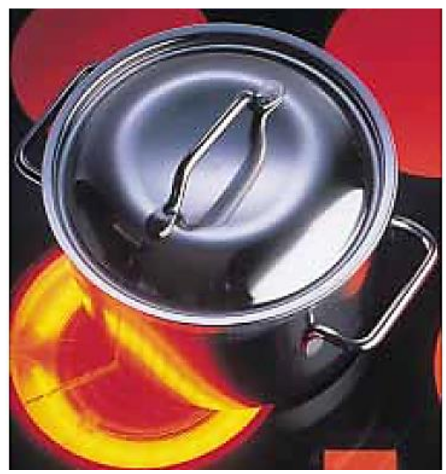


Fig. 4: (a) In an electric guitar, a vibrating string induces an emf in a pickup coil. (b) The circles beneath the metallic strings of this electric guitar detect the notes being played and send this information through an amplifier and into speakers.

Induction Heater

This electric range cooks food on the basis of the principle of induction. An oscillating current is passed through a coil placed below the cooking surface, which is made of a special glass. The current produces an oscillating magnetic field, which induces a current in the cooking utensil. Because the cooking utensil has some electrical resistance, the electrical energy associated with the induced current is transformed to internal energy, causing the utensil and its contents to become hot.



Motional Emf

In the above examples, we considered cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time. In this section we describe what is called motional emf, which is the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length *l* shown in Figure 1 is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. The electrons in the conductor experience a force $\boldsymbol{F_B} = q\boldsymbol{V} \times \boldsymbol{B}$ that is directed along the length *l*, perpendicular to both V and **B**. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there,

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leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced inside the conductor. The charges accumulate at both ends until the downward magnetic force qvB is balanced by the upward electric force qE. At this point, electrons stop moving. The condition for equilibrium requires that

$$qE = qvB \quad or \quad E = vB$$

The electric field produced in the conductor (once the electrons stop moving and E is constant) is related to the potential difference across the ends of the conductor according to the relationship $\Delta V = El$. Thus,

$$\Delta V = El = Blv$$

Where the upper end is at a higher electric potential than the lower end. **Thus, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.** If the direction of the motion is reversed, the polarity of the potential difference also is reversed.

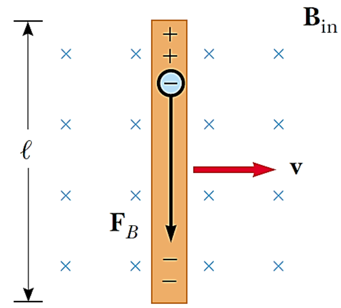

Fig 1: A straight electrical conductor of length *l* moving with a velocity v through a uniform magnetic field B directed perpendicular to v.

A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating how a changing magnetic flux causes an induced current in a closed circuit. Consider a circuit consisting of a conducting bar of length sliding along two fixed parallel conducting rails, as shown in Figure 2a.

For simplicity, we assume that the bar has zero resistance and that the stationary part of the circuit has a resistance R. A uniform and constant magnetic field **B** is applied perpendicular to the plane of the circuit. As the bar is pulled to the right with a velocity **v**, under the influence of an applied force $\mathbf{F}_{app}$, free charges in the bar experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. In this case, the rate of change of magnetic flux through the loop and the corresponding induced motional emf across the moving bar are proportional to the change in area of the loop. As we shall see, if the bar is pulled to the right with a constant velocity, the work done by the applied force appears as internal energy in the resistor R.

Because the area enclosed by the circuit at any instant is x, where x is the width of the circuit at any instant, the magnetic flux through that area is

$$\Phi_B = Blx$$

Using Faraday's law, and noting that x changes with time at a rate *dx/dt=v*, we find that the induced motional emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv \quad (1)$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R} \quad (2)$$

The equivalent circuit diagram for this example is shown in Figure 2b.

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Let us examine the system using energy considerations. Because no battery is in the circuit, we might wonder about the origin of the induced current and the electrical energy in the system. We can understand the source of this current and energy by noting that the applied force does work on the conducting bar, thereby moving charges through a magnetic field. Their movement through the field causes the charges to move along the bar with some average drift velocity, and hence a current is established. Because energy must be conserved, the work done by the applied force on the bar during some time interval must equal the electrical energy supplied by the induced emf during that same interval. Furthermore, if the bar moves with constant speed, the work done on it must equal the energy delivered to the resistor during this time interval.

As it moves through the uniform magnetic field **B**, the bar experiences a magnetic force $\mathbf{F}_B$ of magnitude *IlB.* The direction of this force is opposite the motion of the bar, to the left in Figure 2a. Because the bar moves with constant velocity, the applied force must be equal in magnitude and opposite in direction to the magnetic force, or to the right in Figure 2a. (If $\mathbf{F}_B$ acted in the direction of motion, it would cause the bar to accelerate. Such a situation would violate the principle of conservation of energy.) Using Equation 2 and the fact that $F_{app} = IlB$, we find that the power delivered by the applied force is

$$p = F_{app}v = (IlB)v = \frac{B^2 l^2 v^2}{R} = \frac{\varepsilon^2}{R} \qquad (3)$$

We see that this power is equal to the rate at which energy is delivered to the resistor I^2R, as we would expect. It is also equal to the power supplied by the motional emf. This example is a clear demonstration of the conversion of mechanical energy first to electrical energy and finally to internal energy in the resistor.

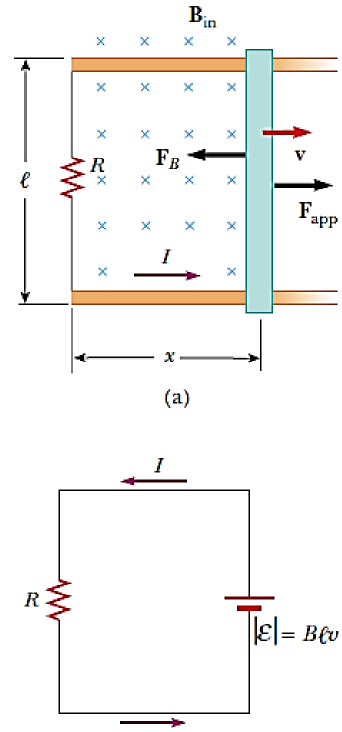


Fig 2: (a) A conducting bar sliding with a velocity v along two conducting rails under the action of an applied force F_{app}. The magnetic force F_B opposes the motion, and a counterclockwise current *I* is induced in the loop. (b)

Self-Inductance Definition: The amount of induced emf in a solenoid in the response of unit change of current in that same coil is called the self-inductance of that solenoid. Its unit is henry (H).

$$\frac{\varepsilon}{\frac{dI}{dt}} = L$$

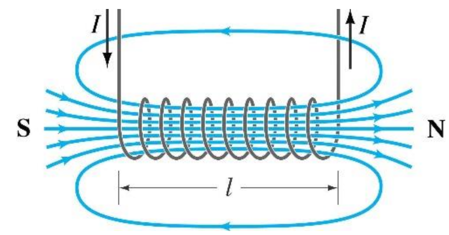
Mutual Inductance definition: The amount of induced emf in a coil in the response of unit change of current in the coil near to it, is called the mutual inductance of that solenoid. Its unit is henry (H).

$$\frac{\varepsilon}{\frac{dI}{dt}} = M$$

Self-Inductance of a Solenoid

The magnetic flux produced in the inner core of the solenoid is equal to:

$$\Phi = B.A$$



Where: Φ is the magnetic flux, B is the flux density, and A is the area.

If the inner core of a long solenoid coil with N number of turns per meter length is hollow, “air cored”, then the magnetic induction within its core will be given as:

$$B = \mu_o H = \mu_o \frac{N.I}{\ell}$$

Then by substituting these expressions in the first equation above for Inductance will give us:

$$L = N\frac{\Phi}{I} = N\frac{B.A}{I} = N\frac{\mu_o.N.I}{\ell.I}.A$$

By cancelling out and grouping together like terms, then the final equation for the coefficient of self-inductance for an air cored coil (solenoid) is given as:

$$L = \mu_o \frac{N^2.A}{\ell}$$

- Where:
- L is in Henries
- μ_o is the Permeability of Free Space ($4.\pi.10^{-7}$ TmA^{-1})
- N is the Number of turns
- A is the Inner Core Area (πr^2) in m^2
- l is the length of the Coil in meters

As the inductance of a coil is due to the magnetic flux around it, the stronger the magnetic flux for a given value of current the greater will be the inductance. So a coil of many turns will have a higher inductance value than one of only a few turns and therefore, the equation above will give inductance L as being proportional to the number of turns squared N^2.

Home work: The self-inductance of a coil of 400 turns is 8 mH. What is the magnetic flux through the coil when the current is 5×10^{-8} amp?

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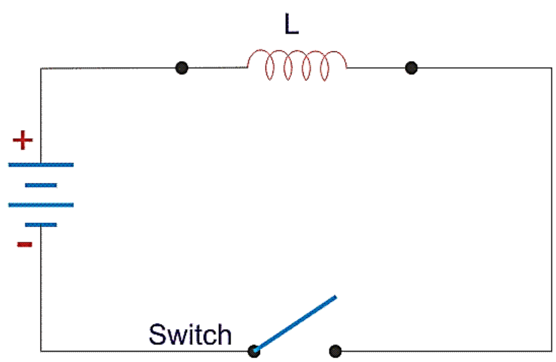
Energy stored in a magnetic field

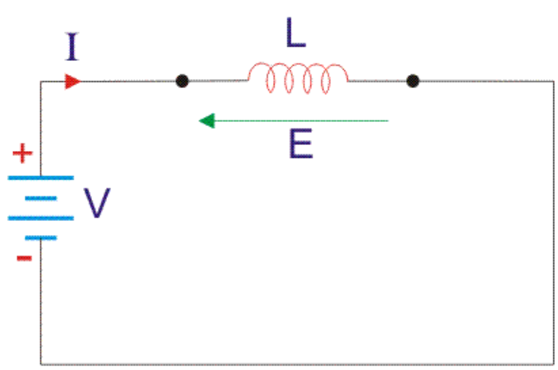
Magnetic field can be of permanent magnet or electro-magnet. Both magnetic fields store some energy. Permanent magnet always creates the magnetic flux and it does not vary upon the other external factors. But electromagnet creates its variable magnetic fields based on how much current it carries. The dimension of this electro-magnet is responsible to create the strength the magnetic field and hence the energy stored in this electromagnet.
First we consider the magnetic field is due to electromagnet i.e. a coil of several no. turns. This coil or inductor is carrying current I when it is connected across a battery or voltage source through a switch.



Suppose battery voltage is V volts, value of inductor is L Henry, and current I will flow at steady state. When the switch is ON, a current will flow from zero to its steady value. But due to self-induction a induced voltage appears which is

$$E = -L\frac{dI}{dt}$$

This E always in the opposite direction of the rate of change of current.



Now here the energy or work done due to this current passing through this inductor is U. As the current starts from its zero value and flowing against the induced emf E, the energy will grow up gradually from zero value to U.

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$$dU = W.dt,$$

Where W is the small power and W = – E.I
So, the energy stored in the inductor is given by

$$dU = W.dt = -E.Idt = L\frac{dI}{dt}.Idt = LIdI$$

Now integrate the energy from 0 to its final value.

$$U = \int_0^U dU = \int_0^I LIdI = \frac{1}{2}LI^2$$

Again,

$$L = \frac{\mu_0 N^2 A}{l}$$

as per dimension of the coil, where N is the number of turns of the coil, A is the effective cross-sectional area of the coil and l is the effective length of the coil.

Again,

$$I = \frac{H.l}{N}$$

Where, H is the magnetizing force, N is the number of turns of the coil and *l* is the effective length of the coil.

$$I = \frac{B.l}{\mu_0 . N}$$

Now putting expression of L and I in equation of U, we get new expression i.e.

$$U = \frac{\frac{\mu_0 N^2 A}{l} . \frac{B.l}{\mu_0 . N}}{2} = \frac{B^2 Al}{2\mu_0}$$

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