



Understanding Time Complexity

Complexity of Algorithms



- The complexity of an algorithm is the function which gives the running time and/or space in terms of the input size.
- In order to compare algorithms, we must have some criteria to measure the efficiency of a algorithm.
- Suppose M is an algorithm, and suppose n is the size of the input data.
- The **TIME** and **SPACE** used by the algorithm M are the two main measures for the efficiency of M .
- The **TIME** is measured by counting the number of key operation-in sorting and searching algorithms. (the # of comparison)
- The **SPACE** is measured by counting the maximum of memory needed by the algorithm



“IN Computer Science, time complexity describes the amount of time that takes to run an algorithm or Code.”





How to calculate Time Complexity ?

“Time complexity is calculated by counting the Number of Operation , which takes a fixed amount of time to perform.”





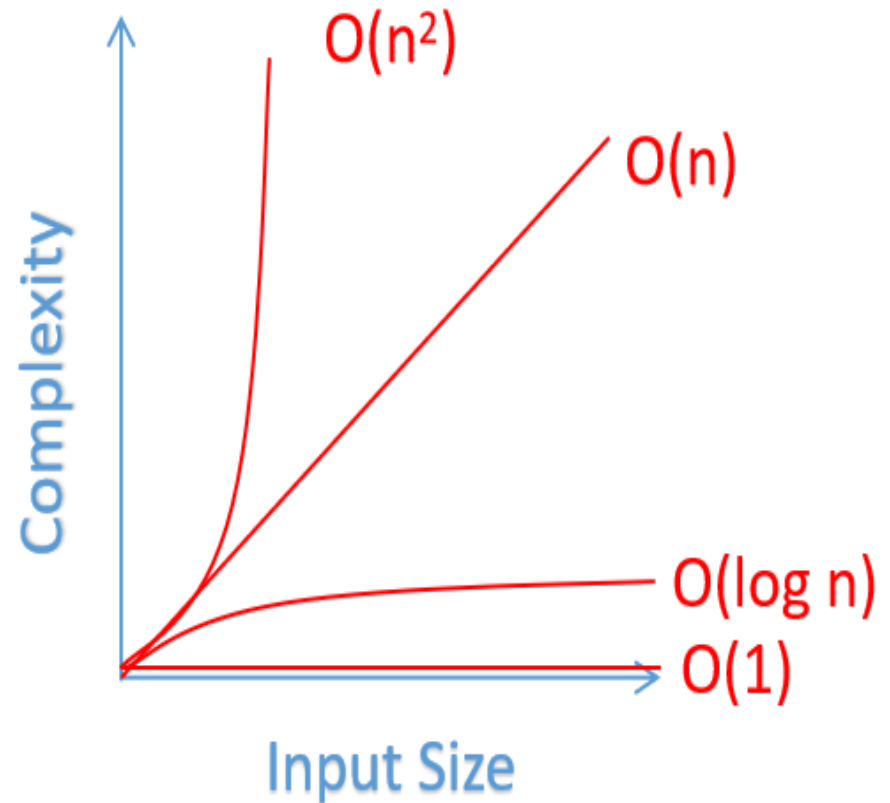
Big O Notation

“Big O notation is a mathematical notation that describes the limiting behavior of a function when the argument tends towards a particular value or infinity.”





- **$O(1)$ Constant Time complexity**
- **$O(N)$ Linear Time Complexity**
- **$O(N^2)$ Quadratic Time Complexity**
- **$O(N \log N)$ Logarithmic Time Complexity**





- Input (in single operation)
- Output (in single operation)

```
#include <iostream>
using namespace std;

int main() {

    int a,b;
    cin >> a >> b;
    //0(1+1)=0(1) constant operation.
    int sum = a+b;
    cout<< sum <<endl;
    return 0;
}
```



Linear Time Complexity - $O(N)$

- While doing operation with *Single* for or while loop.
- Total Complexity($O(N)$)
- In 1 Sec $1e7$ Or maximum $1e8$ Iteration can be done.

```
#include<bits/stdc++.h>
using namespace std;
int main()
{
    int t;
    cin >> t;
        int n;
    cin >> n;
    int arr[n];
    for(int i=0;i<n;i++){
        cin >> arr[i];
    }    //  $O(N)$  *here N is Number of
element
    sort(arr,arr+n);
    for(int i=0;i<n;i++){
        cout << i << " ";
    }    //  $O(N)$  *here N is Number of
element
    cout << endl;
}
```




Quadratic Time Complexity – $O(N^2)$

- “If Two nested loop used and iterate through all the elements. Then complexity is $O(n^2)$.”
- Explanation : iteration for the inner loop : $(n-1)*(n-2)*\dots*2*1$.
- So the result is $n*(n-1)$ which is $O(n^2)$.

```
#include<bits/stdc++.h>
using namespace std;
int main()
{
    int n;
    cin >> n;
    int arr[n];
    for(int i=0;i<n;i++) cin >> arr[i];
    //Bubble sort
    for( int i = 0; i < n; i++ )
    {
        for(int j = 0; j < (n-i-1); j++) {
            if(arr[j] > arr[j+1]){
                swap( arr[j] , arr[j+1] );
            }
        }
    }
    for(auto u : arr){
        cout << u << " ";
    }
    cout << endl;
}
```



Logarithmic Time Complexity – $O(\log N)$

Per Operation mid divides into half Value. That means

$$n/2 + n/4 + n/8 + n/16 + \dots$$

$$= n (1/2 + 1/4 + 1/8 + \dots)$$

$$= \log(n) \quad // \log_2(y) = x, 2^x$$

$$= y;$$

So, Time Complexity of Binary search is $O(\log(n))$

```
int main()
{ // Binary Search
  int n , v[n]; cin >> n;
  for(int i=0; i<n; i++){
    cin >> v[i];
  }
  int lo=0,hi=n-1,mid,fnd;
  cin >> fnd;
  while(hi>=lo){
    mid=(hi+lo)/2;
    if(v[mid] == fnd) {
      cout << mid << endl;
      break;
    }
    if(v[mid] <= fnd){
      lo=mid+1;    }
    else
    {
      hi = mid-1;
    }  }
}
```

Flowcharts

