Some information

Real life example of time dilation: Muons are unstable particles that have the same charge as an electron, but a mass 207 times more than an electron. Muons have a half-life of $\Delta t_p = 2.2\ \mu s$ when measured in a reference frame at rest with respect to them (muons). Relative to an observer on earth, muons should have a lifetime of $\gamma \Delta t_p = 1.56 \times 10^{-5} s$ for V=0.99C. A CERN experiment measured lifetime in agreement with the predictions of relativity.

Some points need to know:

- Electrodynamics -- Motion of charge (moving charge)
- Moving charge radiate energy in the form of electromagnetic wave
- A unique medium, called ether was suggested for the propagation of em wave.
- Maxwell describes as, changing E-field and M-field in the medium produce strain in the medium which propagates as em wave.
- In electrodynamics, motion of charge is vital, which is not vital in classical physics
- Static charge→ electric field
- Moving charge → both electric & magnetic field
- State of a charge (rest or motion) depends on the choice of inertial frame. Hence for electrodynamics principle of relativity (all inertial frame are equivalent) does not held good
- Laws of classical mechanics is invariant in relativity
- Then which frame is the preferred frame for a charge?
- To ascertain the velocity of charge definitely a preferred frame is to be conceived. Ether frame is taken as absolute rest frame.
- Ether expand everywhere, in is immovable. The velocity of charge implies velocity w.r.to ether frame only
- There should be a current when something moving through it. But this is not happens in the case of ether.
- To establish the ether hypothesis→Ether drag hypothesis
- We have three choice:
  1. Relativity principle good for mechanics but not valid for electrodynamics
  2. Relativity principle valid for both mechanics and electrodynamics. Maxwell's equation needs modification.
  3. Relativity principle valid for electrodynamics, Maxwell's equation valid but mechanics need modification

Route Dependence of Proper Time

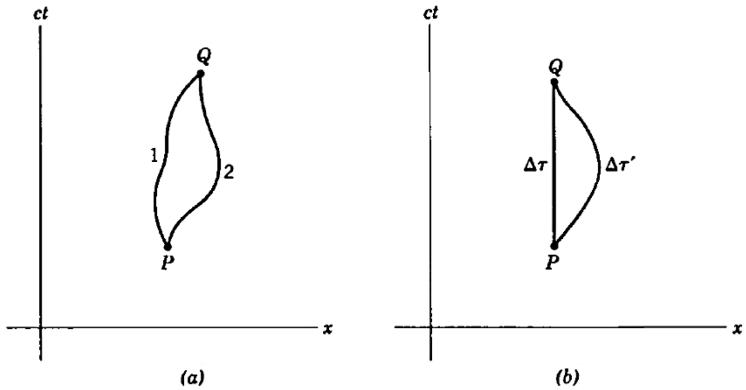

Fig. 1

Consider a space-time diagram (Fig. la) which is relevant to our problem. We can connect events P and Q by different possible world lines (1 and 2 in Fig. la). We are not surprised that the distance traveled between P and Q (the odometer reading) depends on the route we take. It is also true, however, that the time recorded by the traveling clocks depends on the route taken. Let us illustrate this result directly. The time recorded by a clock attached to the object tracing out a world line is the proper time. We have seen that the relationship between the proper time τ and the time t is $d\tau = dt\sqrt{1-\frac{v^2}{C^2}}$. For motion in one space dimension we can write this also as $d\tau = \sqrt{dt^2 - \frac{dx^2}{C^2}}$. The elapsed proper time between events P and Q then is simply

$$\Delta\tau = \int_P^Q \sqrt{dt^2 - \frac{dx^2}{C^2}}$$

Where we integrate along the (world line) path from P to Q. Consider now the particular case, Fig. 1b, in which one world line represents a clock at rest on the x-axis; this gives a vertical line. Suppose now that we have a second path by which an identical second clock is taken from P to Q. Such a clock moves away from the first one and then returns to it, the clocks being coincident at P and at Q. The elapsed proper time along the first world line is

$$\Delta\tau = \int_P^Q \sqrt{dt^2 - \frac{dx^2}{C^2}} = \int_P^Q \sqrt{dt^2 - 0} \; = \int_P^Q dt \; = t_Q - t_P$$

for dx is zero along this path and the proper time coincides with the time interval, t_Q - t_P, recorded by the rest clocks. Along the second world line, however, the elapsed proper time is

$$\Delta\tau' = \int_P^Q \sqrt{dt^2 - \frac{dx^2}{C^2}}$$

$\Delta\tau'$will not equal $\Delta\tau$. In fact, since dx^2 is always positive, we find that $\Delta\tau' < \Delta\tau$.

The clocks will read different times when brought back together, the traveling clock running behind (recording a smaller time difference than) the stay-at-home clock. We should note here that the x-t frame is an inertial frame. The motion of the traveling clock is represented in this frame by a curved world line, for this clock undergoes accelerated motion rather than motion with uniform velocity. It could not return to the stationary clock, for example, without reversing its velocity. The special theory of relativity can predict the behavior of accelerated objects as long as, in the formulation of the physical laws, we take the view of the inertial (unaccelerated) observer. This is what we have done so far. A frame attached to the clock traveling along its round-trip path would not be an inertial frame. We could reformulate the laws of physics so that they have the same form for accelerated (noninertial) observers—this is the program of general relativity theory—but it is unnecessary to do so to explain the twin paradox. All we wish to point out here is that the situation is not symmetrical with respect to the clocks (or twins); one is always in a single inertial frame and the other is not.

Space-Time Diagram of the "Twin Paradox"

In our earlier discussions of time dilation, we spoke of "moving clocks running slow." What is meant by that phrase is that a clock moving at a constant velocity u relative to an inertial frame containing synchronized clocks will be found to run slow by the factor $\sqrt{1-\frac{v^2}{c^2}}$ when timed by those clocks. That is, to time a clock moving at constant velocity relative to an inertial frame, we need at least two synchronized clocks in that frame. We found this result to be reciprocal in that a single S'-clock is timed as running slow by the many S-clocks, and a single S-clock is timed as running slow by the many S'-clocks.

The situation in the twin paradox is different. If the traveling twin traveled always at a constant speed in a straight line, he would never get back home. And each twin would indeed claim that the other's clock runs slow compared to the synchronized clocks in his own frame. To get back home—that is, to make a round trip—the traveling twin would have to change his velocity. What we wish to compare in the case of the twin paradox is a single moving clock with a single clock at rest. To do this we must bring the clocks into coincidence twice—they must come back together again. It is not the idea that we regard one clock as moving and the other at rest that leads to the different clock readings, for if each of two observers seems to the other to be moving at constant speed in a straight line they cannot absolutely assert who is moving and who is not. Instead, it is because one clock has changed its velocity and the other has not that makes the situation unsymmetrical.

Now you may ask how the twins can tell who has changed his velocity. This is clear-cut. Each twin can carry an accelerator. If he changes his speed or the direction of his motion, the acceleration will be detected. We may not be aware of an airplane's motion, or a train's motion, if it is one of uniform velocity; but let it move in a curve, rise and fall, speed up or slow down and we are our own accelerometer as we get thrown around. Our twin on the ground watching us does not experience these feelings his accelerometer registers nothing. Hence, we can tell the twins apart by the fact that the one who makes the round-trip experiences and records accelerations whereas the stay-at-home does not.

A numerical example, suggested by Darwin, is helpful in fixing the ideas. We imagine that, on New Year's Day, Bob leaves his twin brother Dave, who is at rest on a space ship floating in free space. Bob, in another space ship, fires rockets that get him moving at a speed of 0.8C relative to Dave and by his own clock travels away for three years. He then fires more powerful rockets that exactly reverse his motion and gets to Dave after another three years by his clock. By firing rockets a third time he comes to rest beside Dave and compares clock readings. Bob's clock says he has been away for six years, but Dave's clock says ten years have elapsed. Let us see how this comes about.

First, we can simplify matters by ignoring the effect of the accelerations on the traveling clock. Bob can turn off his clock during the three acceleration periods, for example. The error thereby introduced can be made very small compared to the total time of the trip, for we can make the trip as far and as long as we wish without changing the acceleration intervals. It is the total time that is at issue here in any case. We do not destroy the asymmetry, for even in the ideal simplification of Fig. B-2 (where the world lines are straight lines rather than curved ones) Dave is always in one inertial frame whereas Bob is definitely in two different inertial frames one going out (0.8c) and another coming in (- 0.8c).

Let the space ships be equipped with identical clocks which send out light signals at one-year intervals. Dave receives the signals arriving from Bob's clock and records them against the annual signals of his own clock; likewise, Bob receives the signals from Dave's clock and records them against the annual signals of his clock.

In Fig. 2, Dave's world line is straight along the ct-axis:, he is at x = 0 and we mark off ten years (in terms of ct), a dot corresponding to the annual New Year's Day signal of his clock. Bob's world line at first is a straight line inclined to the ct-axis, corresponding to a ct'-axis of a frame moving at + 0.8c relative to Dave's frame. We mark off three years (in terms of *ct'*), a dot corresponding to the annual New Year's Day signal of his clock. After three of Bob's years, he switches to another inertial frame whose world line is a straight line inclined to the ct-axis, corresponding to the ct''-axis of a frame moving at -0.8c relative to Dave's frame. We mark off three years (in terms of ct''), a dot corresponding to the annual New Year's Day signal of his clock. Note the dilation of the time interval of Bob's clock compared to Dave's.

Let us now draw in the light signals from Bob's clock. From each dot on Bob's world line we draw a straight line inclined 45° to the axes (corresponding to a light signal of speed c) headed back to Dave on the line at x = 0. There are six signals, the last one emitted when Bob returns home to Dave. Likewise, the signals from Dave's clock are straight lines, from each dot on Dave's world line, inclined 45° to the axes and headed out to Bob's spaceship. We see that there are ten signals, the last one emitted when Bob returns home to Dave.

How can we confirm this space-time diagram numerically? Simply by the Doppler effect. As the clocks recede from each other, the frequency of their signals is reduced from the proper frequency by the Doppler effect. In this case the Doppler factor is

$$\sqrt{\frac{c-v}{c+v}} = \sqrt{\frac{c-0.8c}{c+0.8c}} = \sqrt{\frac{0.2}{1.8}} = \frac{1}{3}$$

Hence, Bob receives the first signal from Dave after three of his years, just as he is turning back. Similarly, Dave receives messages from Bob on the way out once every three of his years, receiving three signals in nine years. As the clocks approach one another, the frequency of their signals is increased from the proper frequency by the Doppler effect. In this case the Doppler factor is

$$\sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{c+0.8c}{c-0.8c}} = \sqrt{\frac{1.8}{0.2}} = \frac{3}{1}$$

Thus, Bob receives nine signals from Dave in his three-year return journey. Altogether, Bob receives ten signals from Dave. Similarly, Dave receives three signals from Bob in the last year before Bob is home. Altogether, Dave receives six signals from Bob.

There is no disagreement about the signals: Bob sends six and Dave receives six; Dave sends ten and Bob receives ten. Everything works out, each seeing the correct Doppler shift of the other's clock and each agreeing to the number of signals that the other sent. The different total times recorded by the twins corresponds to the fact that Dave sees Bob recede for nine years and return in one year, although Bob both receded for three of his years and returned for three of his years. Dave's records will show that he received signals at a slow rate for nine years and at a rapid rate for one year. Bob's records will show that he received signals at a slow rate for three years and at a rapid rate for another three years. The essential asymmetry is thereby revealed by a Doppler effect analysis. When Bob and Dave compare records, they will agree that Dave's clock recorded ten years and Bob's recorded only six. Ten years have passed for Dave during Bob's six-year round trip.

Will Bob really be four years younger than his twin brother?

Since for the word "clock" we could have substituted any periodic natural phenomena, such as heart-beat or pulse rate, the answer is yes. We might say that Bob lived at a slower rate then Dave during his trip, his bodily functions proceeding at the same slower rate as his physical clock. Biological clocks behave in this respect the same as physical clocks. There is no evidence that there is any difference in the physics of organic processes and the physics of the inorganic materials involved in these processes. If motion affects the rate of a physical clock, we expect it to effect a biological clock in the same way.

It is of interest to note the public acceptance of the idea that human life processes can be slowed down by refrigeration, so that a corresponding different aging of twins can be achieved by temperature differences. What is paradoxical about the relativistic case, in which the different aging is due to the difference in motion, is that since (uniform) motion is relative, the situation appears (incorrectly) to by symmetrical. But, just as the temperature differences are real, measurable, and agreed upon by the twins in the foregoing example, so are the differences in motion real, measurable, and agreed upon in the relativistic case—the changing of inertial frames, that is the accelerations, are not symmetrical. The results are absolutely agreed upon.

Although there is no need to invoke general relativity theory in explaining the twin paradox, the student may wonder what the outcome of the analysis would be if we knew how to deal with accelerated reference frames. We could then use Bob's space ship as our reference frame, so that Bob is the stay-at-home, and it would be Dave who, in this frame, makes the round-trip space journey. We would find that we must have a gravitational field in this frame to account for the accelerations that Bob feels and the fact that Dave feels no accelerations even though he makes a round trip. If, as required in general relativity, we then compute the frequency shifts of light in this gravitational field, we come to the same conclusion as in special relativity.

An Experimental Test

The experiments accessible to us are not those of spacemen traveling at speeds near that of light; they are instead radioactive nuclei whose change in ticking (photon decay rate) at different speeds can be measured to an extremely high accuracy. A radioactive source of gamma ray photons can be tuned to resonance with an absorber of such photons to within a very sharp frequency interval (Mossbauer Effect). A source (radioactive iron-57 nuclei) mounted at the center of a rotor and a resonant absorber on the perimeter are used, the measurements being made as a function of the angular velocity of the rotor. The experiment can be analyzed in the inertial frame of the source using special relativity or in the reference frame of the accelerated absorber using general relativity. The measurements may be regarded as a transverse Doppler effect or a time dilation produced by gravitation, each expressing the same fact that the clock that is accelerated is slowed down compared to the clock at rest. One twin stays at home; the other literally makes a round trip. The results of these experiments show that a group of radio-active nuclei on the perimeter of the turning rotor undergo fewer decays than an identical set of radioactive nuclei at rest at the center of the rotor. The round-trip twin ages less than his stay-at-home brother and, within the limits of experimental error, by exactly the amount predicted by relativity theory.

Gravitational Time Dilation

A common equation used to determine gravitational time dilation is derived from the Schwarzschild metric, which describes space-time in the vicinity of a non-rotating massive spherically symmetric object. The equation is

$$t = \frac{t_0}{\sqrt{1 - \frac{GM}{rc^2}}} = \frac{t_0}{\sqrt{1 - \frac{r_s}{r}}}$$

where

- t_0 is the proper time between events A and B for a slow-ticking observer within the gravitational field,
- t is the coordinate time between events A and B for a fast-ticking observer at an arbitrarily large distance from the massive object (this assumes the fast-ticking observer is using Schwarzschild coordinates, a coordinate system where a clock at infinite distance from the massive sphere would tick at one second per second of coordinate time, while closer clocks would tick at less than that rate),
- G is the gravitational constant,
- M is the mass of the object creating the gravitational field,
- r is the radial coordinate of the observer (which is analogous to the classical distance from the center of the object, but is actually a Schwarzschild coordinate),
- c is the speed of light, and
- $r_s = \frac{GM}{c^2}$ is the Schwarzschild radius of M.

To illustrate then, without accounting for the effects of rotation, proximity to Earth's gravitational well will cause a clock on the planet's surface to accumulate around 0.0219 fewer seconds over a period of one year than would a distant observer's clock. In comparison, a clock on the surface of the sun will accumulate around 66.4 fewer seconds in one year.