PH501 · ADVANCED NUCLEAR PHYSICS

Lecture 3 — Cross sections and reciprocity

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Reading: Satchler §2.6, §2.17, §2.18.8–11

1. From kinematics to probability

Lecture 2 answered *where* the products can go. This lecture answers *how often* they actually get there. The quantity that encodes that probability is the cross section.

2. Cross section

A beam of flux F (particles per unit area per second) hits a target of areal density n_t (nuclei per unit area). A detector at angle θ, covering solid angle ΔΩ, records R counts per second. The cross section is defined by

$$R = F \cdot n_t \cdot \sigma \qquad \textbf{(1)}$$

σ has dimensions of area (units: barn, 1 b = 10^{-24} cm^2 = 10^{-28} m^2). It is *not* the geometric area of the nucleus; it is an effective area that encodes the probability of the reaction occurring.

3. Differential cross section

Usually we measure the rate into a particular solid angle ΔΩ at angle θ. The differential cross section is

$$d\sigma/d\Omega = (1 / F\, n_t) \cdot \Delta R/\Delta\Omega \qquad \textbf{(2)}$$

Units: mb/sr. Integrating over all solid angle recovers the total: $\sigma = \int (d\sigma/d\Omega)\, d\Omega$.

Quantum-mechanically, $d\sigma/d\Omega = |f(\theta)|^2$, where $f(\theta)$ is the scattering amplitude from the asymptotic wavefunction $\psi \to e^{ikz} + f(\theta)\, e^{ikr}/r$.

4. Rutherford scattering

Pure Coulomb ($V = Z_1Z_2e^2/r$, no nuclear force). Classical orbit is a hyperbola: impact parameter $b = (a/2) \cot(\theta/2)$, where $a = Z_1Z_2e^2/2E$. The cross section $d\sigma = b|db/d\theta|\, d\Omega$ gives

$$d\sigma/d\Omega|_{\mathrm{Ruth}} = (Z_1Z_2e^2 / 4E)^2 / \sin^4(\theta/2)$$

The quantum result for $1/r$ is identical. Diverges at θ → 0 (long-range Coulomb). Any deviation from Rutherford = nuclear effects present. Used as normalisation standard.

5. Double-differential cross section (DDX)

When ejectiles span a continuum of energies (not just discrete states), we resolve in both angle *and* energy:

$$d^2\sigma \,/\, d\Omega\, dE \quad (3)$$

Units: mb/(sr·MeV). A DDX spectrum at a fixed angle shows the full energy distribution of outgoing particles. It is the modern workhorse observable for intermediate-energy reactions, because it displays the reaction mechanisms directly:

Region of spectrum	Mechanism	Signature
Low E_{out}, smooth hump	compound (evaporation)	Maxwellian shape, no angle dependence
Intermediate E_{out}	pre-equilibrium	bridge between compound and direct
High E_{out}, near beam energy	direct (elastic, transfer, break-up)	forward-peaked, discrete or quasi-elastic peak

Example: ^{27}Al(d, px) at E_d = 99.6 MeV measured at 10°–100°. The INC (intranuclear cascade) calculation reproduces the evaporation region and the continuum shape well; it underestimates the direct peak near $E_{out} \approx 100$ MeV at backward angles, where elastic and transfer contributions (not included in INC) dominate.

6. Elastic vs reaction cross section

The total cross section splits into two parts:

$$\sigma_{tot} = \sigma_{el} + \sigma_R$$

σ_{el} is elastic scattering (same particles out, no internal change). σ_R is the reaction cross section: everything else (inelastic, transfer, knock-out, capture, compound). In partial-wave language (Lecture 4), σ_R is related to the *absorption* of flux from the elastic channel, which is why the optical potential needs an imaginary part.

7. The reciprocity theorem

For any reaction $a + A \rightarrow b + B$, the inverse $b + B \rightarrow a + A$ is related by detailed balance:

$$k_a^2\, (2J_a+1)(2J_A+1)\, d\sigma(a\rightarrow b)/d\Omega = k_b^2\, (2J_b+1)(2J_B+1)\, d\sigma(b\rightarrow a)/d\Omega \quad (4)$$

Here k_a, k_b are the CM wavenumbers and J the spins. This holds for both direct and compound reactions. Practical use: if one direction is hard to measure (e.g. a radioactive beam), measure the other and convert.

8. Cross sections and reaction mechanism

Observable	Direct reaction	Compound reaction
σ(E) excitation fn.	structured, resonances on smooth bg.	smooth, Ericson fluctuations
dσ/dΩ	forward-peaked, oscillatory (ℓ-dependent)	symmetric about 90°, 1/sin θ
$d^2\sigma/d\Omega dE$	peaks at high E_{out}	Maxwellian at low E_{out}

The cross section, in each of its forms, is the experimental handle that separates mechanisms and connects to nuclear structure (Lectures 4–8).

Next: Lecture 4

We now have the observable (cross section) and its relation to experiment. What we need next is the theory: how does ψ determine $f(\theta)$? Lecture 4 sets up scattering theory, partial waves, phase shifts, and introduces the optical model.

References

- Satchler, *Introduction to Nuclear Reactions*, 2nd ed. (1990): §2.6, §2.17, §2.18.8–11.
- Bertulani, *Nuclear Physics in a Nutshell* (2007): ch. 10.
- Krane, *Introductory Nuclear Physics* (1988): ch. 11.