

## PH501 · ADVANCED NUCLEAR PHYSICS

**Lecture 4 — Scattering theory and the optical model**

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Reading: Satchler §3.1–3.6, §4.5

**1. The Schrödinger equation**

Last lecture we have learned  $d\sigma/d\Omega = |f(\theta)|^2$ . Now we need  $f(\theta)$ . Two-body problem in the CM frame with a central potential  $V(r)$ :

$$[-(\hbar^2/2\mu)\nabla^2 + V(r)]\psi = E\psi \quad (1)$$

$\mu = m_a m_A / (m_a + m_A)$  is the reduced mass,  $E = T_{CM}$ .

**2. The scattering wavefunction**

Before the nucleus the wave is a plane wave (the beam). After interacting it picks up an outgoing spherical component (what the detector sees). So the boundary condition far from the potential is

$$\psi \rightarrow e^{i^*kz^*} + f(\theta) e^{i^*kr^*}/r \quad (2)$$

$f(\theta)$  is the scattering amplitude. The whole problem: given  $V(r)$ , find  $f(\theta)$ .

**3. Partial-wave expansion**

Instead of tracking individual trajectories, QM decomposes the beam into angular-momentum components. Each  $\ell$  corresponds approximately to one classical impact parameter  $b \approx \ell/k$ . For a central potential,  $\ell$  is conserved, so we expand:

$$\psi = \sum_{\ell} (2\ell+1) i^{\ell} (u_{\ell}(r)/kr) P_{\ell}(\cos \theta) \quad (3)$$

Each  $u_{\ell}(r)$  satisfies a radial equation with effective potential  $V(r) + \hbar^2\ell(\ell+1)/2\mu r^2$ . Low  $\ell$  = head-on. Large  $\ell$  interacts only weakly with the nuclear potential;  $\delta_{\ell} \approx 0$  and the sum converges.

**4. Phase shifts**

The potential shifts the phase of the outgoing wave relative to the free particle:

$$u_{\ell}(r) \rightarrow \sin(kr - \ell\pi/2 + \delta_{\ell}) \quad (4)$$

Think of it as a delay or advance: an attractive potential ( $\delta_{\ell} > 0$ ) pulls the wave inward so it arrives at the detector earlier in phase. Matching (4) with the boundary conditions gives

$$f(\theta) = (1/2ik) \sum_{\ell} (2\ell+1) (e^{2i^*\delta_{\ell}} - 1) P_{\ell}(\cos \theta) \quad (5)$$

$\delta_{\ell} > 0$ : attractive.  $\delta_{\ell} < 0$ : repulsive.  $\delta_{\ell} = 0$ : no interaction.  $\delta_{\ell} = \pi/2$ : resonance (maximum contribution to  $\sigma_{el}$ ).

**5. Elastic cross section**

$$\sigma_{el} = (4\pi/k^2) \sum_{\ell} (2\ell+1) \sin^2 \delta_{\ell} \quad (6)$$

Worked example:  $\delta_0 = 45^\circ$ ,  $\delta_1 = 20^\circ$ ,  $k = 1.0 \text{ fm}^{-1}$ .

$$\ell = 0: (2 \cdot 0 + 1) \sin^2 45^\circ = 0.500$$

$$\ell = 1: (2 \cdot 1 + 1) \sin^2 20^\circ = 0.351$$

$$\sigma_{\text{el}} = 4\pi \times 0.851 = 10.69 \text{ fm}^2 \approx 1069 \text{ mb}$$

If only  $\ell = 0$  contributes  $\rightarrow$  low-energy (s-wave) scattering. If  $\delta_\ell = 0$  for all  $\ell \rightarrow$  no interaction ( $\sigma = 0$ ).

## 6. Reaction cross section

Define the S-matrix element  $S_\ell \equiv \eta_\ell = e^{2i\delta_\ell}$ . Then

$$\sigma_R = (\pi/k^2) \sum_\ell (2\ell+1)(1 - |S_\ell|^2) \quad (7)$$

$|S_\ell| = 1$  means no absorption at that  $\ell$ .  $|S_\ell| < 1$  means flux is removed.  $|S_\ell| = 0$  is the black-disk limit.

## 7. The problem: real potentials conserve flux

A real  $V(r)$  gives a Hermitian Hamiltonian. Probability current is conserved, so  $|S_\ell| = |e^{2i\delta_\ell}| = 1$  for every  $\ell$ . Substituting into (7):  $\sigma_R = 0$ . No flux leaves the elastic channel. But experiments show  $\sigma_R > 0$ . Where does the missing flux go? Into reaction channels. To describe that absorption, the potential must be complex.

## 8. The optical model

$$U(r) = V(r) - iW(r) \quad (8)$$

$V(r)$  refracts (elastic scattering).  $W(r)$  absorbs (reactions). The name: like light through cloudy glass. With complex  $\delta_\ell$ ,  $|S_\ell| = e^{-2\beta_\ell} < 1$  and  $\sigma_R > 0$ .

## 9. Woods-Saxon potential

$$f(r) = 1 / (1 + \exp[(r - R)/a]) \quad (9)$$

$R = r_0 A^{1/3}$  (nuclear radius,  $r_0 \approx 1.25 \text{ fm}$ ),  $a \approx 0.65 \text{ fm}$  (surface diffuseness).

Term	Shape	What it does
$Vf(r)$	volume Woods-Saxon	bulk attraction; sets elastic scattering
$-iW_V f(r)$	volume absorption	absorbs flux throughout interior
$-iW_S df/dr$	surface absorption	absorbs at surface (dominant at low $E$ )
$V_{so} (1/r) df/dr \ell$ -s	spin-orbit	splits $j = \ell \pm 1/2$ ; polarisation data

Typical values:  $V \approx 50 \text{ MeV}$ ,  $W \approx 5\text{--}15 \text{ MeV}$ . Parameters fitted to elastic angular distributions.

## 10. What the optical model gives us

Solve (1) with complex  $U(r)$  numerically. Out come: (i) elastic  $d\sigma/d\Omega$  — compare with data to fix parameters, (ii)  $\sigma_R$  from (7), (iii) the distorted waves  $\chi_{\alpha^+}$ ,  $\chi_{\beta^-}$ , obtained by solving  $[E - K - U]\chi = 0$  with the fitted potential. These distorted waves are the input to the DWBA transition amplitude in L5.

## Next: Lecture 5

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We have  $f(\theta)$  for elastic scattering. For *reactions* we need  $T_{\beta\alpha}$ . L5 derives the DWBA:  $T_{\beta\alpha}^{\text{DW}} = \langle \chi_{\beta}^{-} | W_{\beta} | \psi_{\alpha} \chi_{\alpha}^{+} \rangle$ .

## References

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- Satchler, *Introduction to Nuclear Reactions*, 2nd ed.: §3.1–3.6, §4.5.
- Bertulani, *Nuclear Physics in a Nutshell*. Relevant scattering/optical-model chapter.
- Krane, *Introductory Nuclear Physics*: ch. 11.