Electric Current

Electron Theory of Conductivity

The classical model of electrical conduction in metals that was first proposed by Paul Drude in 1900. This model leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals. Although the Drude model described here does have limitations, it nevertheless introduces concepts that are still applied in more elaborate treatments.

Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called conduction electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, gain mobility when the free atoms condense into a solid. In the absence of an electric field, the conduction electrons move in random directions through the conductor with average speeds of the order of 10^6 m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an **electron gas**. There is no current through the conductor in the absence of an electric field because the drift velocity of the free electrons is **zero**. That is, on the average, just as many electrons move in one direction as in the opposite direction, and so there is no net flow of charge.

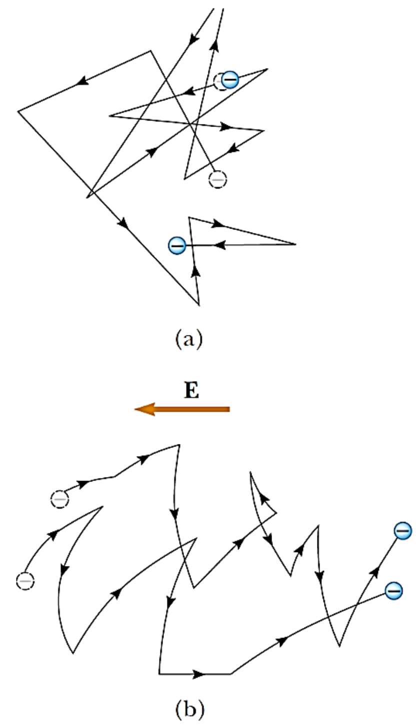

Figure 1: (a) A schematic diagram of the random motion of two charge carriers in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of the charge carriers in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carriers have a drift velocity.

This situation changes when an electric field is applied. Now, in addition to undergoing the random motion just described, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed v_d that is much smaller (typically 10^{-4} m/s) than their average speed between collisions (typically 10^6 m/s).

Figure 1 provides a basic description of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 1a). An electric field E modifies the random motion and causes the electrons to drift in a direction opposite that of E (Fig. 1b). The slight curvature in the paths shown in Figure 1b results from the acceleration of the electrons between collisions, which is caused by the applied field.

In our model, we assume that the motion of an electron after a collision is independent of its motion before the collision. We also assume that the excess energy acquired by the electrons in the electric field is lost to the atoms of the conductor when the electrons and atoms collide. The energy given up to the atoms increases their vibrational energy, and this causes the temperature of the conductor to increase. The temperature increase of a conductor due to resistance is utilized in electric toasters and other familiar appliances.

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We will now derive an expression for the drift velocity. When a free electron of mass m_e and charge $q=(-e)$ is subjected to an electric field $\boldsymbol{E}$, it experiences a force $\boldsymbol{F}=q\boldsymbol{E}$. Because $\sum \boldsymbol{F} = m_e \boldsymbol{a}$, we conclude that the acceleration of the electron is

$$\boldsymbol{a} = \frac{q\boldsymbol{E}}{m_e} \tag{1}$$

This acceleration, which occurs for only a short time between collisions, enables the electron to acquire a small drift velocity. If t is the time since the last collision and v_i is the electron's initial velocity the instant after that collision, then the velocity of the electron after a time t is

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}t = \boldsymbol{v}_i + \frac{q\boldsymbol{E}}{m_e}t \tag{2}$$

We now take the average value of $\boldsymbol{v}_f$ over all possible times t and all possible values of $\boldsymbol{v}_i$. If we assume that the initial velocities are randomly distributed over all possible values, we see that the average value of $\boldsymbol{v}_i$ is zero. The term $\frac{q\boldsymbol{E}}{m_e}t$ is the velocity added by the field during one trip between atoms. If the electron starts with zero velocity, then the average value of the second term of Equation 2 is $\frac{q\boldsymbol{E}}{m_e}\tau$ where, τ is the *average time interval between successive collisions*. Because the average value of $\boldsymbol{v}_f$ is equal to the drift velocity, we have

$$\overline{\boldsymbol{v}}_f = \boldsymbol{v}_d = \frac{q\boldsymbol{E}}{m_e}\tau \quad \text{Drift velocity} \tag{3}$$

We can relate this expression for drift velocity to the current in the conductor. Substituting this into equation of the current density we find

$$\boldsymbol{J} = nq\boldsymbol{v}_d = nq\frac{q\boldsymbol{E}}{m_e}\tau = \frac{nq^2\boldsymbol{E}}{m_e}\tau$$

Where n is the number of charge carriers per unit volume. Comparing this expression with Ohm's law, $J = \sigma E$ we obtain the following relationships for conductivity and resistivity:

$$\sigma = \frac{nq^2}{m_e}\tau \quad \text{Conductivity}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau} \quad \text{Resistivity}$$

According to this classical model, conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm's law.

The average time between collisions τ is related to the average distance between collisions l (that is, the mean free path) and the average speed $\overline{\boldsymbol{v}}$ through the expression

$$\tau = \frac{l}{\overline{\boldsymbol{v}}}$$

Problem 1: The 12-gauge copper wire in a building has a cross-sectional area of 3.31×10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

(a) Using the data and results from Example 27.1 and the classical model of electron conduction, estimate the average time between collisions for electrons in household copper wiring. (b) Assuming that the average speed for free electrons in copper is 1.6×10^6 m/s and using the result from part (a), calculate the mean free path for electrons in copper.

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Where $\rho = 1.7 \times 10^{-8}\ \Omega m$ for copper and the carrier density is $n = 8.49 \times 10^{28}$ electrons/m3 for the wire.

Solution (a): Substitution of these values into the expression above gives

$$\tau = \frac{m_e}{nq^2\rho} = \frac{9 \times 10^{-31}}{8.49 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}}\ S$$

$$= 2.5 \times 10^{-14}\ S$$

Solution (b): We know mean free path is

$$l = \tau\bar{\boldsymbol{v}} = 2.5 \times 10^{-14}\ \times 1.6 \times 10^{6}\ m$$

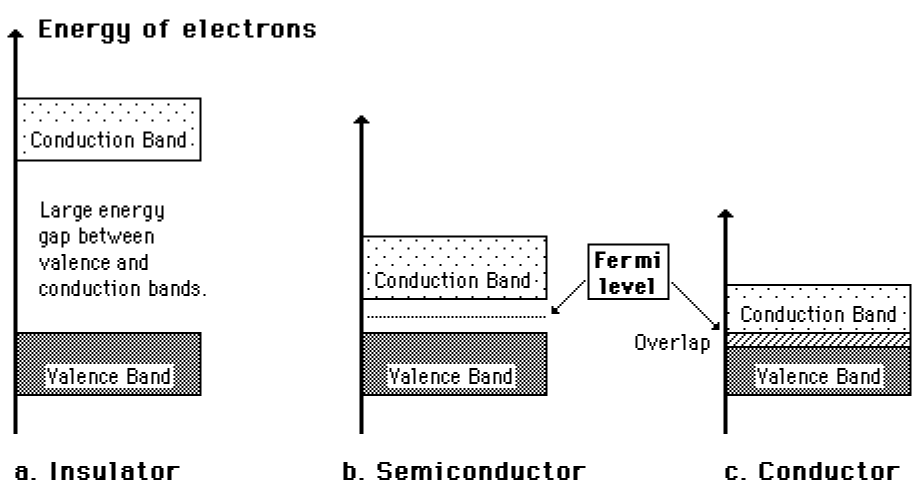
$$= 4.0 \times 10^{-8}\ m$$

Which is equivalent to 40 nm (compared with atomic spacing of about 0.2 nm). Thus, although the time between collisions is very short, an electron in the wire travels about 200 atomic spacing between collisions.

Band Theory of Solids

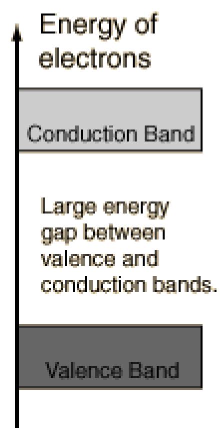
A useful way to visualize the difference between conductors, insulators and semiconductors is to plot the available energies for electrons in the materials. Instead of having discrete energies as in the case of free atoms, the available energy states form bands. Key to the conduction process is whether or not there are electrons in the conduction band. In insulators the electrons in the valence band are separated by a large gap from the conduction band, in conductors like metals the valence band overlaps the conduction band, and in semiconductors there is a small enough gap between the valence and conduction bands that thermal or other excitations can bridge the gap. With such a small gap, the presence of a small percentage of a doping material can increase conductivity dramatically.

An important parameter in the band theory is the Fermi level, the top of the available electron energy levels at zero kelvin. The position of the Fermi level with the relation to the conduction band is a crucial factor in determining electrical properties.



Insulator Energy Bands

Most solid substances are insulators, and in terms of the band theory of solids this implies that there is a large forbidden gap between the energies of the valence electrons and the energy at which the electrons can move freely through the material (the conduction band).



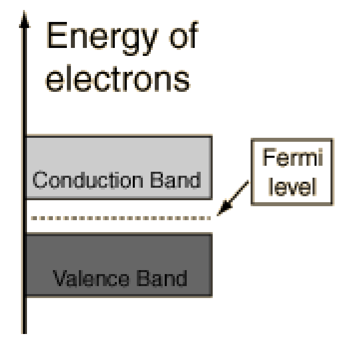
a. Insulator

Glass is an insulating material which may be transparent to visible light for reasons closely correlated with its nature as an electrical insulator. The visible light photons do not have enough quantum energy to bridge the band gap and get the electrons up to an available energy level in the conduction band. The visible properties of glass can also give some insight into the effects of "doping" on the properties of solids. A very small percentage of impurity atoms in the glass can give it color by providing specific available energy levels which absorb certain colors of visible light. The ruby mineral (corundum) is aluminum oxide with a small amount (about 0.05%) of chromium which gives it its characteristic pink or red color by absorbing green and blue light.

While the doping of insulators can dramatically change their optical properties, it is not enough to overcome the large band gap to make them good conductors of electricity. However, the doping of semiconductors has a much more dramatic effect on their electrical conductivity and is the basis for solid state electronics.

Semiconductor Energy Bands

For intrinsic semiconductors like silicon and germanium, the Fermi level is essentially halfway between the valence and conduction bands. Although no conduction occurs at 0 K, at higher temperatures a finite number of electrons can reach the conduction band and provide some current. In doped semiconductors, extra energy levels are added.

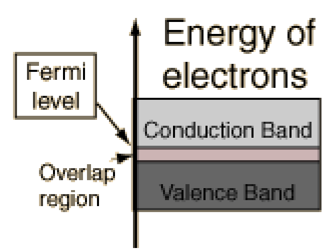


b. Semiconductor

The increase in conductivity with temperature can be modeled in terms of the Fermi function, which allows one to calculate the population of the conduction band.

Conductor Energy Bands

In terms of the band theory of solids, metals are unique as good conductors of electricity. This can be seen to be a result of their valence electrons being essentially free. In the band theory, this is depicted as an overlap of the valence band and the conduction band so that at least a fraction of the valence electrons can move through the material.



c. Conductor

Why bands and band gaps occur

The electrons of a single, isolated atom occupy atomic orbitals each of which has a discrete energy level. When two or more atoms join together to form into a molecule, their atomic orbitals overlap. The Pauli Exclusion Principle dictates that no two electrons can have the same quantum numbers in a molecule. So if two identical atoms combine to form a diatomic molecule, each atomic orbital splits into two molecular orbitals of different energy, allowing the electrons in the former atomic orbitals to occupy the new orbital structure without any having the same energy.

Similarly if a large number N of identical atoms come together to form a solid, such as a crystal lattice, the atoms' atomic orbitals overlap. Since the Pauli exclusion principle dictates that no two electrons in the solid have the same quantum numbers, each atomic orbital splits into N discrete molecular orbitals, each with a different energy. Since the number of atoms in a macroscopic piece of solid is a very large number ($N \sim 10^{22}$) the number of orbitals is very large and thus they are very closely spaced in energy (of the order of 10−22 eV). The energy of adjacent levels is so close together that they can be considered as a continuum, an energy band.

This formation of bands is mostly a feature of the outermost electrons (valence electrons) in the atom, which are the ones involved in chemical bonding and electrical conductivity. The inner electron orbitals do not overlap to a significant degree, so their bands are very narrow.

Band gaps are essentially leftover ranges of energy not covered by any band, a result of the finite widths of the energy bands. The bands have different widths, with the widths depending upon the degree of overlap in the atomic orbitals from which they arise. Two adjacent bands may simply not be wide enough to fully cover the range of energy. For example, the bands associated with core orbitals (such as 1s electrons) are extremely narrow due to the small overlap between adjacent atoms. As a result, there tend to be large band gaps between the core bands. Higher bands involve comparatively larger orbitals with more overlap, becoming progressively wider at higher energies so that there are no band gaps at higher energies.

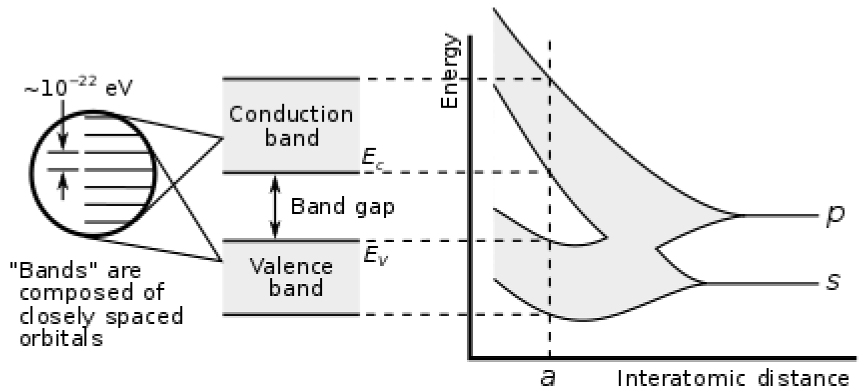


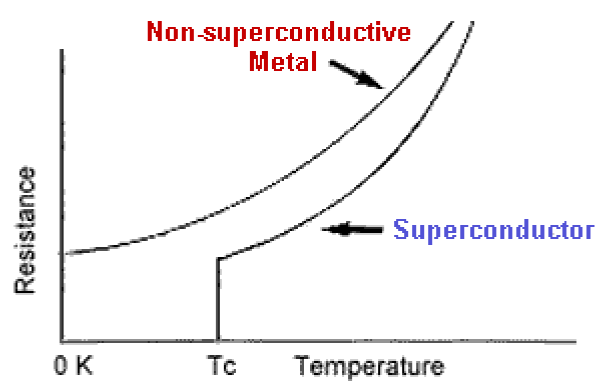
Figure showing how electronic band structure comes about by the hypothetical example of a large number of carbon atoms being brought together to form a diamond crystal. The graph *(right)* shows the energy levels as a function of the spacing between atoms. When the atoms are far apart *(right side of graph)* each atom has valence atomic orbitals p and s which have the same energy. However, when the atoms come closer together their orbitals begin to overlap. Due to the Pauli Exclusion Principle each atomic orbital splits into *N* molecular orbitals each with a different energy, where *N* is the number of atoms in the crystal. Since *N* is such a large number, adjacent orbitals are extremely close together in energy so the orbitals can be considered a continuous energy band. *a* is the atomic spacing in an actual crystal of diamond. At that spacing the orbitals form two bands, called the valence and conduction bands, with a 5.5 eV band gap between them

Comparison between Conductors, Semiconductors and Insulators

Parameter	Conductor	Semiconductor	Insulator
Forbidden energy gap	Not exist	Small (1 eV)	Large (>5 eV)
Conductivity	High (10^{-7} mho/m)	Medium (10^{-7} to 10^{-13} mho/m)	Very Low (10^{-3} mho/m) Almost negligible.
Resistivity	Low	Moderate	High
Flow of current	Due to movement of free electrons.	Due to movement of electrons and holes.	Almost negligible but only due to free electrons.
Temperature coefficient of resistance	Positive	Negative	Negative
Charge carriers in conduction band	Completely filled	Partially filled	Completely vacant
Charge carriers in valence band	Almost vacant	Partially filled	Completely filled
Example	Copper, Aluminium, graphite etc.	Silicon, Germanium, arsenic etc.	Paper, rubber, glass, plastic etc.
Applications	Conducting wires, Transformers, in electrical cords etc.	Diodes, transistors, optocouplers etc.	Sports equipment, home appliances etc.

Superconductors

The electrical resistivity of many metals and alloys drops suddenly to zero when the specimen is cooled to a sufficiently low temperature, often a temperature in the liquid helium range. The phenomena of the reduction in the value of electrical resistivity to zero is known as superconductivity and the materials which show this property are called the superconductors. Examples of superconductors are Nb_3Ge, Nb_3Al, K_3C_6O etc.



This phenomena was observed first by Heike Kamerlingh Onnes in 1911 in mercury. He observed a sudden decrease in resistivity when pure mercury was cooled down below 4.2 k. The resistivity of mercury vanish completely below 4.2 k.

Transition/Critical temperature: The temperature at which the specimen undergoes a phase transition from a state of normal electrical resistivity to a superconducting state is known as transition of critical temperature. It is denoted by T_C

Basic Properties of a Superconductor:

For a material to be considered as a superconductor it has to exhibit two distinctive/basic properties:

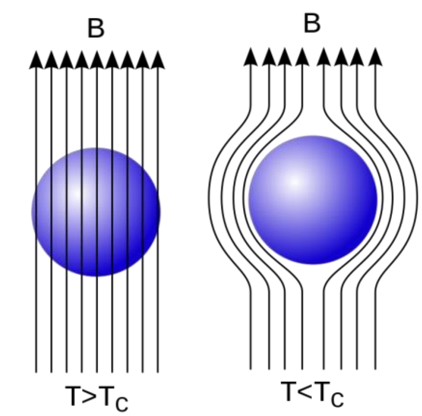
i) It has no resistivity for all temperature below the critical temperature of that material. i.e. $\rho = 0 \;\; for\, T < T_C.$
ii) It has no magnetic induction i.e. B=0 inside the superconductor (i.e. perfect diamagnetic below T_C)

Persistent Current:

If a superconductor has the form of a ring, a current can be induced in the ring by the electromagnetic induction. We have simply to cool the ring in a magnetic field from a temperature above the critical temperature T_C to below T_C and then to remove the field. Now it has been observed that this current continue to persist with undiminished strength for years. In a typical experiment a led ring could carry an induced current of several hundred amperes for a year without any change, such current is called persistent current.

Meissner Effect:

Meissner and Ochsenfeld found that, if a superconductor is cooled in a magnetic field to below the transition temperature, then at the transition the lines of induction B are pushed out. This phenomenon is called the Meissner effect. Meissner effect shows that a bulk superconductor behaves in an applied external field B_a as if inside the specimen, B=0. We obtain a particularly useful form of this result if we limit ourselves to long thin specimens with long axes parallel to B_a. Now the demagnetizing field contribution to magnetic induction B will be negligible. Hence



$$B = B_a + 4\pi M = 0$$

$$B_a = -4\pi M$$

Where B_a is the applied field and M is the magnetization of the specimen. Now

$$\frac{M}{B_a} = -\frac{1}{4\pi}$$

$$\chi = -\frac{1}{4\pi}$$

Where $\chi = \frac{M}{B_a}$ is called the magnetic susceptibility, which is –ve in this case. This means that superconductors exhibit perfect diamagnetism.

This important result cannot be derived from the characterization of a superconductor as a mediam of zero resistivity. From the Ohm's law, we can write $\vec{E} = \rho\vec{J}$.

If the resistivity ρ goes to zero while $\vec{J}$ is held infinite, then $\vec{E}$ must be zero. By Maxwell's equation $\frac{d\vec{B}}{dt}$ is proportional to curl **E**, i.e.

$$\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \vec{E}$$

So that zero resistivity implies $\frac{d\vec{B}}{dt} = -\vec{\nabla} \times \vec{E} = -\rho(\vec{\nabla} \times \vec{J}) = 0$

So $\vec{B} = constant$

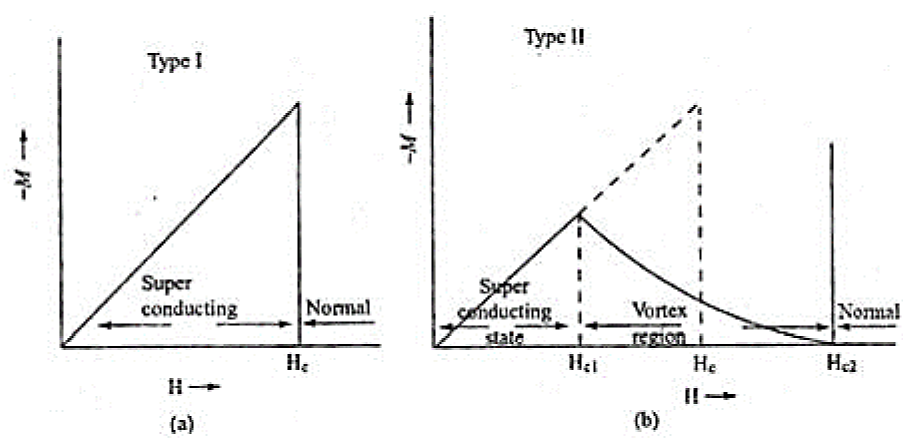
The Meissner effect contradicts this result and suggests that, perfect diamagnetism is an essential property of superconducting state.

A superconductor with little or no magnetic field within it is said to be in the **Meissner state**. The Meissner state breaks down when the applied magnetic field is too strong. Superconductors can be divided into two classes according to how this breakdown occurs.

In **TYPE-I** superconductors, superconductivity is abruptly destroyed when the strength of the applied field rises above a critical value Hc. Depending on the geometry of the sample, one may obtain an intermediate state consisting of a baroque pattern of regions of normal material carrying a magnetic field mixed with regions of superconducting material containing no field.

In **TYPE-II** superconductors, raising the applied field past a critical value Hc_1 leads to a mixed state (also known as the vortex state) in which an increasing amount of magnetic flux penetrates the material, but there remains no resistance to the electric current as long as the current is not too large. At a second critical field strength Hc_2, superconductivity is destroyed. The mixed state is caused by vortices in the electronic superfluid, sometimes called fluxons because the flux carried by these vortices is quantized.

Most pure elemental superconductors, except niobium and carbon nanotubes, are type I, while almost all impure and compound superconductors are type II.



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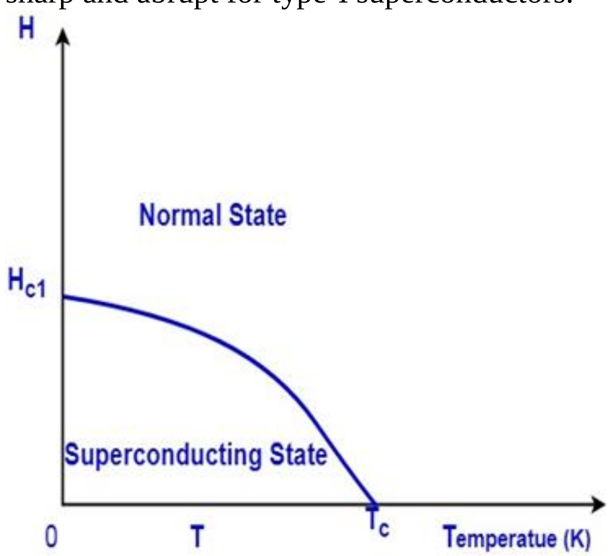
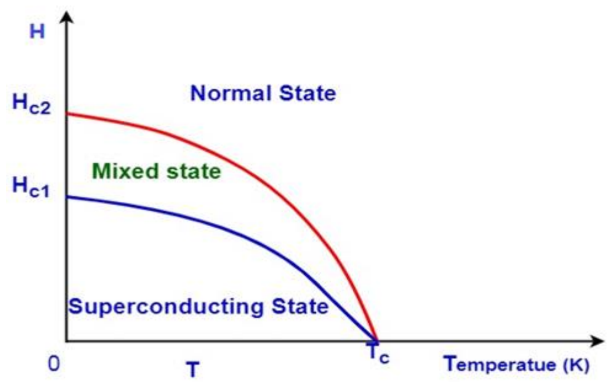
Difference between type-I and type-II superconductors:

Type – I Superconductors	Type – II Superconductors
Low critical temperature (typically in the range of 0K to 10K)	High critical temperature (typically greater than 10K)
Low Critical magnetic field (Typically in the range of 0.0000049 T to 1T)	High Critical magnetic field (Typically greater than 1T)
Perfectly obey the Meissner effect: Magnetic field cannot penetrate inside the material.	Partly obey the Meissner effect but not completely: Magnetic field can penetrate inside the material.
Exhibits single critical magnetic field.	Exhibits two critical magnetic field
Easily lose the superconducting state by low-intensity magnetic field. Therefore, type-I superconductors are also known as soft superconductors.	Does not easily lose the superconducting state by external magnetic field. Therefore, type-II superconductors are also known as hard superconductors.
The transition from a superconducting state to a normal state due to the external magnetic field is sharp and abrupt for type-I superconductors.  Variation in critical magnetic field with temperature for type-I superconductor	The transition from a superconducting state to a normal state due to the external magnetic field is gradually but not shape and abrupt. At lower critical magnetic field (H_{C1}), type-II superconductor starts losing its superconductivity. At upper critical magnetic field (H_{C2}), type-II superconductor completely loses its superconductivity. The state between lower critical magnetic field and upper magnetic field is known as an intermediate state or mixed state.  Variation in critical magnetic field with temperature for type-II superconductor
Due to the low critical magnetic field, type-I superconductors cannot be used for manufacturing electromagnets used for producing strong magnetic field.	Due to the high critical magnetic field, type-II superconductors can be used for manufacturing electromagnets used for producing strong magnetic field.
Type-I superconductors are generally pure metals.	Type-II superconductors are generally alloys and complex oxides of ceramics.

BCS theory can be used to explain the superconductivity of type-I superconductors.	BCS theory cannot be used to explain the superconductivity of type-II superconductors.
These are completely diamagnetic.	These are not completely diamagnetic
No mixed state exists in type-I Superconductors.	A mixed state exists in type-II Superconductors.
Slight impurity does not affect the superconductivity of type-I superconductors.	Slight impurity greatly affects the superconductivity of type-II superconductors.
Due to the low critical magnetic field, type-I superconductors have limited technical applications.	Due to the high critical magnetic field, type-II superconductors have wider technical applications.
Examples: Hg, Pb, Zn,etc.	Examples: NbTi, Nb_3Sn, etc.

Current and Current Density

Electric Current

Whenever there is a net flow of charge through some region, a current is said to exist. To define current more precisely, suppose that the charges are moving perpendicular to a surface of area A, as shown in Figure 1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface**. If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

If the rate at which charge flows varies in time, then the current varies in time; we define the ***instantaneous current*** I as the differential limit of average current:

$$I_{av} = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

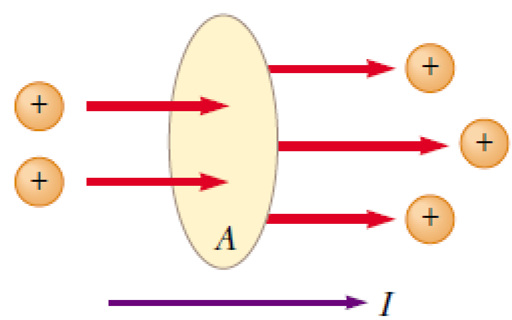


Figure 1: Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

The SI unit of current is the ampere (A):

$$1A = \frac{1C}{1s}$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s. The charges passing through the surface in Figure 1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or aluminum the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, the actual direction of the current is opposite the direction of flow of electrons. However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges.

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. The current in the conductor is zero even if the conductor has an excess of charge on it. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move around the loop and thus creating a current. It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, the mobile charge carriers in a metal are electrons.

Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area A (Fig. 1). The volume of a section of the conductor of length x (the gray region shown in Fig. 1) is $A\Delta x$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is $nA\Delta x$. Therefore, the charge ΔQ in this section is

$$\Delta Q = (\text{number of carriers in section}) \times (\text{charge per carrier}) = (nA\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a speed v_d, the distance they move in a time Δt is $\Delta x = v_d \Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d\Delta t)q$$

If we divide both sides of this equation by Δt, we see that the average current in the conductor is

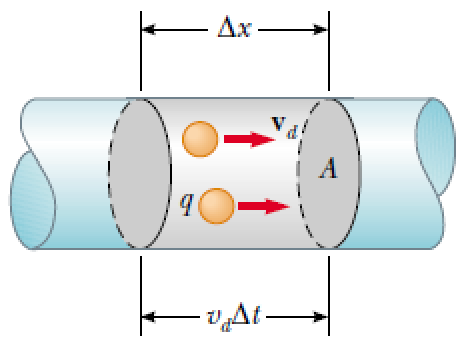


Figure 1: A section of a uniform conductor of cross-sectional area A. The mobile charge carriers move with a speed v_d, and the distance they travel in a time Δt is $\Delta x = v_d\Delta t$.

$$I_{av} = \frac{\Delta Q}{\Delta t} = nAv_d q$$

The speed of the charge carriers v_d is an average speed called the **drift speed**. To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in a direction opposite that of **E**) at the drift velocity $\boldsymbol{v}_d$.

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

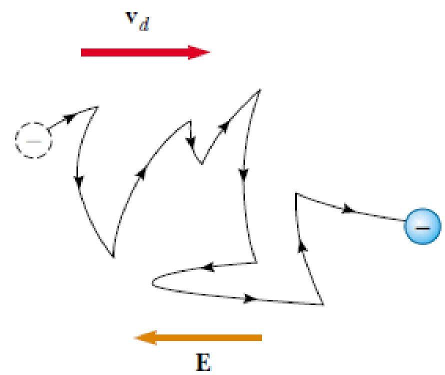


Figure 2: A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor.

Problem 1: The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31$\times 10^{-6}\ m^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

Solution: From the periodic table of the elements we know the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (=1mole) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g/mol}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$\text{n} = \frac{6.02\ \times 10^{23}\ electrons}{7.09\ \text{cm}^3}\ (1.00\ \times 10^{6}\ \text{cm}^3/\text{m}^3)$$
$$= 8.49\ \times\ 10^{28}\ electrons/\text{m}^3$$

We know the drift speed is

$$v_d = \frac{I}{nAq}$$

Where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{10.0}{8.49\ \times\ 10^{28}\ \times 3.31 \times 10^{-6}\ \times 1.6 \times 10^{-19}}\ m/s$$
$$= 2.22\ \times 10^{-4}\ m/s$$

Exercise: If a copper wire carries a current of 80.0 mA, how many electrons flow past a given cross-section of the wire in 10.0 min?

Solution: Try yourself.

Answer: $3.0 \times\ 10^{20}\ electrons$

Resistance and Ohm's Law

We know that no electric field can exist inside a conductor. However, this statement is true only if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are allowed to move.

Charges moving in a conductor produce a current under the action of an electric field, which is maintained by the connection of a battery across the conductor. An electric field can exist in the conductor because the charges in this situation are in motion—that is, this is a **nonelectrostatic** situation.

Consider a conductor of cross-sectional area A carrying a current I. The **current density J** in the conductor is defined as the current per unit area. Because the current the current $I = nAv_d q$, the current density is

$$J = \frac{I}{A} = nv_d q,$$

where J has SI units of A/m^2. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. In general, the current density is a vector quantity:

$$\boldsymbol{J} = n\boldsymbol{v}_d q,$$

From this equation, we see that current density, like current, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$\boldsymbol{J} = \sigma \boldsymbol{E},$$

Where the constant of proportionality σ is called the conductivity of the conductor. Materials that obey this equation are said to follow Ohm's law, named after Georg Simon Ohm. More specifically, Ohm's law states that

for many materials (including most metals), the ratio of the current density to the electric field is a constant $\boldsymbol{\sigma}$ that is independent of the electric field producing the current.

Materials that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be ohmic.

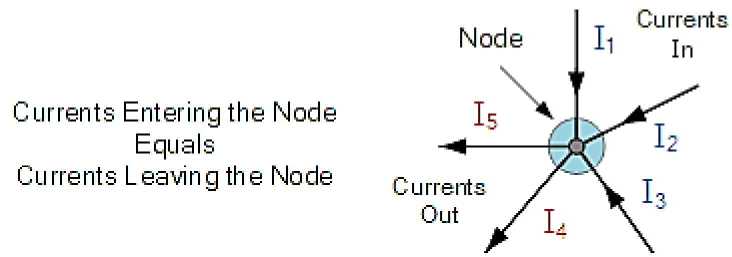
Kirchhoff's Law and its Applications

Kirchhoff's circuit laws are two equalities that deal with the current and potential difference (commonly known as voltage) in the lumped element model of electrical circuits. They were first described in 1845 by German physicist Gustav Kirchhoff.

Kirchhoff's First Law – The Current Law, (KCL)

Kirchhoff's Current Law or KCL, states that the "total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node''.

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero, $I_{(exiting)} + I_{(entering)} = 0$. This idea by Kirchhoff is commonly known as the Conservation of Charge.



$$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$$

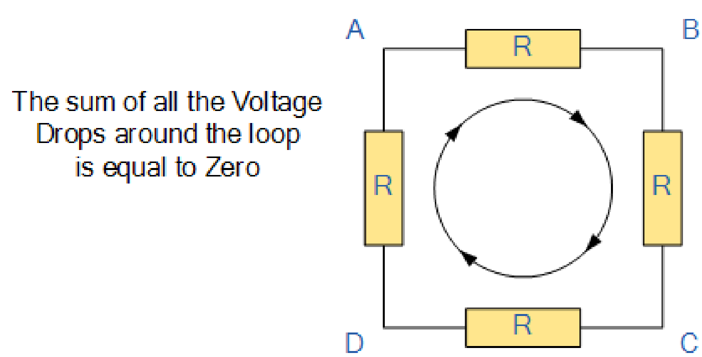
Here, the three currents entering the node, I_1, I_2, I_3 are all positive in value and the two currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

$I_1 + I_2 + I_3 - I_4 - I_5 = 0$

The term Node in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchhoff's current law when analyzing parallel circuits.

Kirchhoff's Second Law – The Voltage Law, (KVL)

Kirchhoffs Voltage Law or KVL, states that "in any closed loop network, the directional sum of the voltage drops in various components in the loop is equal to the directional sum of the e.m.f.'s of the voltage source in the same network". In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchhoff is known as the Conservation of Energy.



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

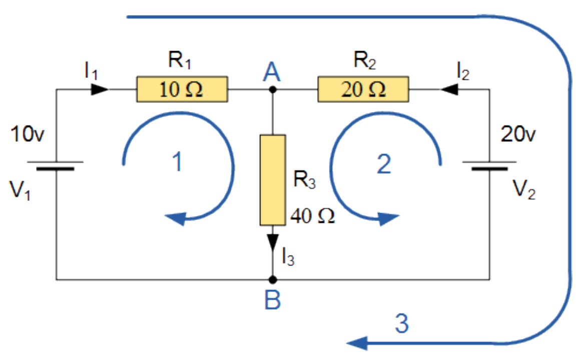
Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchhoff's voltage law when analyzing series circuits.

Common DC Circuit Theory Terms:

When analyzing either DC circuits or AC circuits using Kirchhoff's Circuit Laws a number of definitions and terminologies are used to describe the parts of the circuit being analyzed such as: node, paths, branches, loops and meshes. These terms are used frequently in circuit analysis so it is important to understand them.

- *• Circuit – a circuit is a closed loop conducting path in which an electrical current flows.*
- *• Path – a single line of connecting elements or sources.*
- *• Node – a node is a junction, connection or terminal within a circuit were two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.*
- *• Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.*
- *• Loop – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once.*
- *• Mesh – a mesh is a single open loop that does not have a closed path. There are no components inside a mesh.*

Problem 1: Find the current flowing in the 40Ω Resistor, R_3 also find the voltage drop in this resistor in the circuit below.



Solution: The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using **Kirchhoffs Current Law**, **KCL** the equations are given as:

At node A : $I_1 + I_2 = I_3$

At node B : $I_3 = I_1 + I_2$

Using **Kirchhoffs Voltage Law**, **KVL** the equations are given as:

Loop 1 is given as : $10 = R_1 I_1 + R_3 I_3 = 10I_1 + 40I_3$

Loop 2 is given as : $20 = R_2 I_2 + R_3 I_3 = 20I_2 + 40I_3$

Loop 3 is given as : $10 - 20 = 10I_1 - 20I_2$

As I_3 is the sum of $I_1 + I_2$ we can rewrite the equations as;

Eq. No 1 : $10 = 10I_1 + 40(I_1 + I_2) = 50I_1 + 40I_2$

Eq. No 2 : $20 = 20I_2 + 40(I_1 + I_2) = 40I_1 + 60I_2$

We now have two simultaneous equations that can be reduced to give us the values of I_1 and I_2. Substitution of I_1 in terms of I_2 gives us the value of I_1 as -0.143 Amp

Substitution of I_2 in terms of I_1 gives us the value of I_2 as +0.429 Amp

As : $I_3 = I_1 + I_2$

The current flowing in resistor R_3 is given as : $-0.143 + 0.429 = 0.286$ Amp

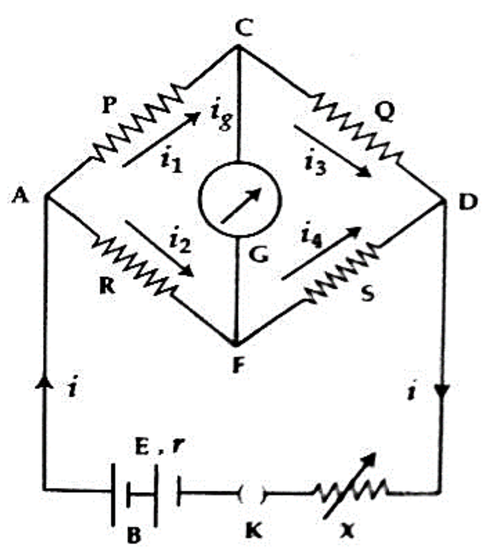
and the voltage across the resistor R_3 is given as : $0.286 \times 40 = 11.44$ volt

The negative sign for I_1 means that the direction of current flow initially chosen was wrong, but never the less still valid. In fact, the 20v battery is charging the 10v battery.

Applications of Kirchhoff's Law

1. Wheatstone bridge principle.

If four junctions are made due to the formation of a closed-loop by connecting four resistors in series and if an electric cell is connected between the two opposite junctions and a galvanometer is connected between the other two opposite junctions then the circuit thus formed is called Wheatstone bridge. The Wheatstone bridge is a circuit which is used to measure correctly an unknown resistance. Wheatstone bridge principle states that when the bridge is impartial, the products of the resistance of the opposite arms are equivalent.



Let the resistance of the galvanometer be G and currents flowing through P, Q, S, and G be respectively i_1, i_2, i_3, i_4, and i_g.

Now, applying Kirchhoff's first law respectively at points C and F. we get,

$$i_1 - i_3 - i_g = 0 \text{ i.e., } i_1 = i_3 + i_g \quad \ldots\ \ldots\ \ldots (1)$$

$$\text{and, } \quad i_2 + i_g - i_4 = 0 \text{ i.e., } i_4 = i_2 + i_g \quad \ldots\ \ldots\ \ldots (2)$$

Again, applying Kirchhoff's second law respectively at closed loops ACFA and CDFC, we get,

$$i_1P + i_gG - i_2R = 0 \quad \ldots\ \ldots\ \ldots (3)$$

$$\text{and, } i_3Q - i_4S - i_gG = 0 \quad \ldots\ \ldots\ \ldots (4)$$

But at balanced condition of the bridge, $i_g = 0$.

So, under this condition, according to equations (1) and (2), $i_1 = i_3$ and $i_4 = i_2$.

According to equations, (3) and (4), $i_1P = i_2R$ … … … (5)

and $i_3Q = i_4S$ … … … (6)

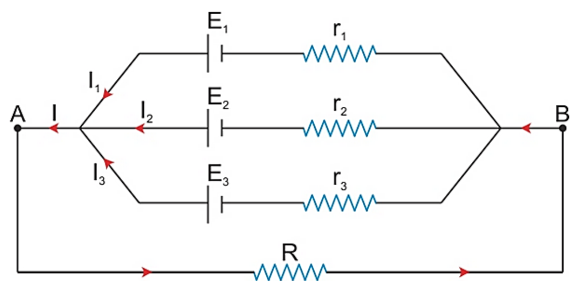
Now dividing equation (5) by equation (6) we get

$i_1P / i_3Q = i_2R / i_4S$; but, $i_1 = i_3$ and $i_4 = i_2$

So, $P / Q = R / S$ … … … (7)

According to the equation (7), at the equilibrium of the Wheatstone bridge, if values of any three resistors are known, then the resistance of the fourth resistor can be determined. It is called the Wheatstone bridge principle for the measurement of resistance.

2. Parallel Combination of Cells



Let us consider e.m.f.'s of three cells are E_1, E_2 and E_3 and their internal resistance are r_1, r_2, and r_3 respectively. They are connected in parallel with a resistor R. Let the current flow due to individual cells are I_1, I_2, and I_3. Applying KCL at a and b points we get

$$I_1 + I_2 + I_3 = I \quad (1)$$

Applying KVL in RAE_1BR, RAE_2BR, RAE_3BR loops we get

$$I_1 r_1 + IR = E_1 \quad (2)$$

$$I_2 r_2 + IR = E_2 \quad (3)$$

$$I_3 r_3 + IR = E_3 \quad (4)$$

Dividing eqn 2, 3, and 4 by r_1, r_2, and r_3 respectively and then summing we get

$$(I_1 + I_2 + I_3) + I\left(\frac{R}{r_1} + \frac{R}{r_2} + \frac{R}{r_3}\right) = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

$$I + I\left(\frac{R}{r_1} + \frac{R}{r_2} + \frac{R}{r_3}\right) = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

$$I\left(1 + \frac{R}{r_1} + \frac{R}{r_2} + \frac{R}{r_3}\right) = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}$$

$$I = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{1 + \frac{R}{r_1} + \frac{R}{r_2} + \frac{R}{r_3}} \quad (5)$$

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If there are n number of cells are connected in this manner then total current will be

$$I = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \cdots \ldots \ldots .. + \frac{E_n}{r_n}}{1 + \frac{R}{r_1} + \frac{R}{r_2} + \frac{R}{r_3} + \cdots \ldots \ldots .. + \frac{R}{r_n}}$$

And if all the cell are of identical e.m.f. and internal resistance r then

$$I = \frac{\frac{nE}{r}}{1 + \frac{nR}{r}}$$

$$I = \frac{nE}{nR + r} \qquad (6)$$

3. Series Combination of Cells

Try yourself.

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