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Faraday's Experiment

To see how an emf can be induced by a changing magnetic field, let us consider a loop of wire connected to a galvanometer, as illustrated in Figure 1. When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right in Figure 1a. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in Figure 1c. When the magnet is held stationary relative to the loop (Fig. 1b), no deflection is observed. Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects. From these observations, we conclude that the loop "knows" that the magnet is moving relative to it because it experiences a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field. These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit! We call such a current an induced current and say that it is produced by an induced emf.

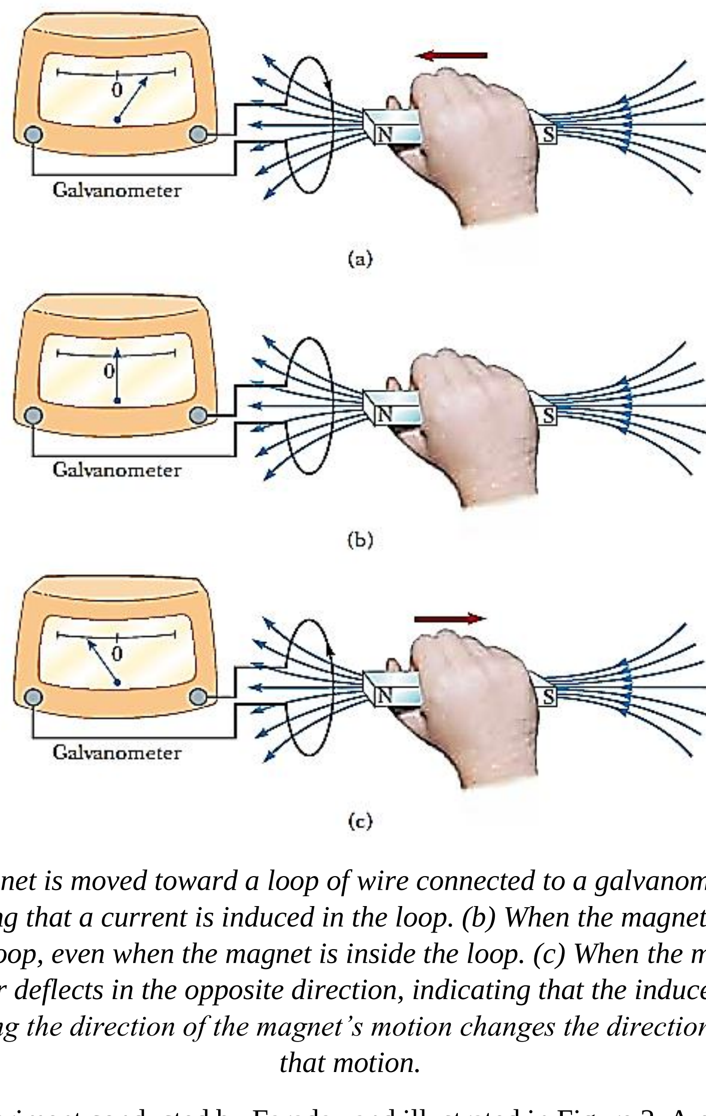


Fig. 1: *(a) When a magnet is moved toward a loop of wire connected to a galvanometer, the galvanometer deflects as shown, indicating that a current is induced in the loop. (b) When the magnet is held stationary, there is no induced current in the loop, even when the magnet is inside the loop. (c) When the magnet is moved away from the loop, the galvanometer deflects in the opposite direction, indicating that the induced current is opposite that shown in part (a). Changing the direction of the magnet's motion changes the direction of the current induced by that motion.*

Now let us describe an experiment conducted by Faraday and illustrated in Figure 2. A primary coil is connected to a switch and a battery. The coil is wrapped around a ring, and a current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a galvanometer. No battery is present in the secondary circuit, and the secondary coil is not connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent. Initially, you might guess that no current is ever detected in the secondary circuit. However, something quite amazing happens when the switch in the primary circuit is either suddenly closed or suddenly opened. At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero. Finally, the galvanometer reads zero when there is either a steady current or no current in the primary circuit.

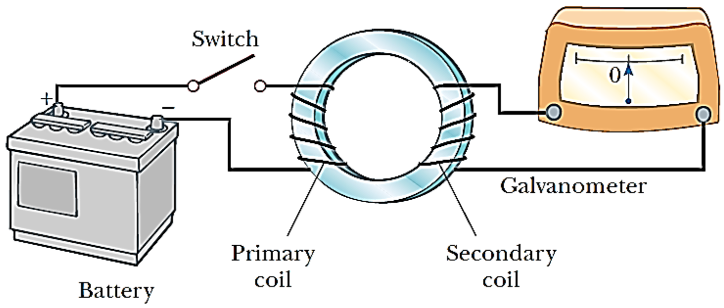


Fig.2: Faraday's experiment. When the switch in the primary circuit is closed, the galvanometer in the secondary circuit deflects momentarily. The emf induced in the secondary circuit is caused by the changing magnetic field through the secondary coil.

The key to understanding what happens in this experiment is to first note that when the switch is closed, the current in the primary circuit produces a magnetic field in the region of the circuit, and it is this magnetic field that penetrates the secondary circuit. Furthermore, when the switch is closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and it is this changing field that induces a current in the secondary circuit.

As a result of these observations, Faraday concluded that an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field. The induced current exists for only a short time while the magnetic field through the secondary coil is changing. Once the magnetic field reaches a steady value, the current in the secondary coil disappears. In effect, the secondary circuit behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the secondary circuit by the changing magnetic field.

Faraday's Law

The experiments shown in Figures 1 and 2 have one thing in common: In each case, an emf is induced in the circuit when the magnetic flux through the circuit changes with time.

In general**, the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit**. This statement, known as **Faraday's law of induction**, can be written

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt} \qquad (1$$

Where $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ is the magnetic flux through the circuit. If the circuit is a coil consisting of N loops all of the same area and if Φ_B is the flux through one loop, an emf is induced in every loop; thus, the total induced emf in the coil is given by the expression

$$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad (2$$

The negative sign in Equations 1 and 2 is of important physical significance, which we shall discuss in Lenz law. Suppose that a loop enclosing an area A lies in a uniform magnetic field B, as shown in Figure 3. The magnetic flux through the loop is equal to $BA\cos\theta$.

Hence, the induced emf can be expressed as

$$\varepsilon = -\frac{d}{dt}(BA\cos\theta) \qquad (3$$

From this expression, we see that an emf can be induced in the circuit in several ways:

• The magnitude of **B** can change with time.
• The area enclosed by the loop can change with time.
• The angle θ between **B** and the normal to the loop can change with time.
• Any combination of the above can occur.

Some Applications of Faraday's Law

The Ground Fault Interrupter (GFI of GFCI)

The ground fault interrupter is an interesting safety device that protects users of electrical appliances against electric shock. Its operation makes use of Faraday's law. In the GFI shown in Figure 3, wire 1 leads from the wall outlet to the appliance to be protected, and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounds the two wires, and a sensing coil is wrapped around part of the ring. Because the currents in the wires are in opposite directions, the net magnetic flux through the sensing coil due to the currents is zero. However, if the return current in wire 2 changes, the net magnetic flux through the sensing coil is no longer zero. (This can happen, for example, if the appliance gets wet, enabling current to leak to ground.) Because household current is alternating (meaning that its direction keeps reversing), the magnetic flux through the sensing coil changes with time, inducing an emf in the coil. This induced emf is used to trigger a circuit breaker, which stops the current before it is able to reach a harmful level.

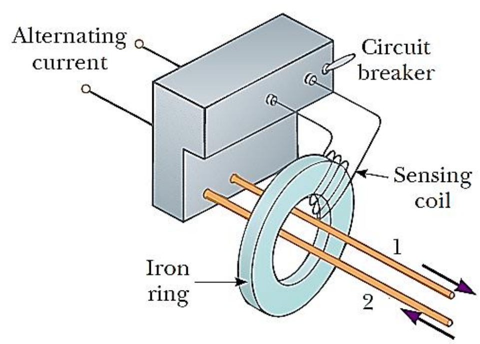


Fig. 3: Essential components of a ground fault interrupter.

Production of Sound in an Electric Guitar

Another interesting application of Faraday's law is the production of sound in an electric guitar (Fig. 4). The coil in this case, called the pickup coil, is placed near the vibrating guitar string, which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the portion of the string nearest Lenz's law the coil. When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces an emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produce the sound waves we hear.

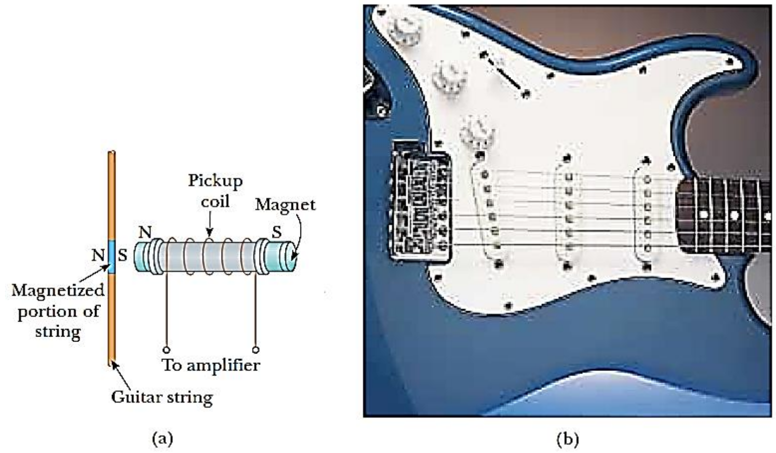
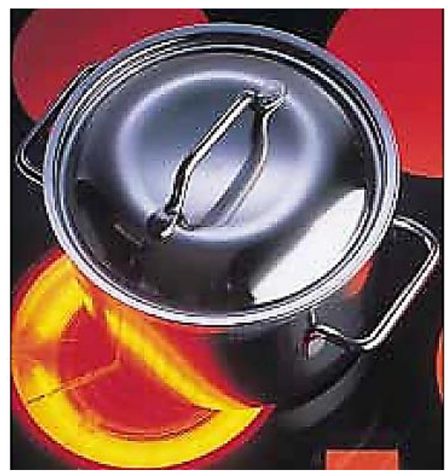


Fig. 4: (a) In an electric guitar, a vibrating string induces an e.m.f in a pickup coil. (b) The circles beneath the metallic strings of this electric guitar detect the notes being played and send this information through an amplifier and into speakers.

Induction Heater

This electric range cooks food on the basis of the principle of induction. An oscillating current is passed through a coil placed below the cooking surface, which is made of a special glass. The current produces an oscillating magnetic field, which induces a current in the cooking utensil. Because the cooking utensil has some electrical resistance, the electrical energy associated with the induced current is transformed to internal energy, causing the utensil and its contents to become hot.



Motional Emf

In the above examples, we considered cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time. In this section we describe what is called motional emf, which is the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length l shown in Figure 1 is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. The electrons in the conductor experience a force $\boldsymbol{F_B} = q\boldsymbol{V} \times \boldsymbol{B}$ that is directed along the length l, perpendicular to both V and **B**. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced inside the conductor. The charges accumulate at both ends until the downward magnetic force qvB is balanced by the upward electric force qE. At this point, electrons stop moving. The condition for equilibrium requires that

$$qE = qvB \quad or \quad E = vB$$

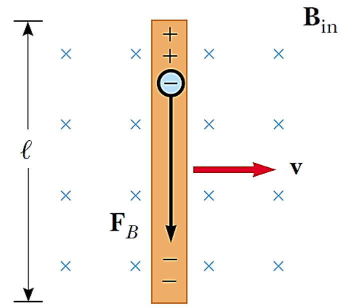


Fig 1: A straight electrical conductor of length *l* moving with a velocity v through a uniform magnetic field B directed perpendicular to v.

The electric field produced in the conductor (once the electrons stop moving and E is constant) is related to the potential difference across the ends of the conductor according to the relationship $\Delta V = El$. Thus,

$$\Delta V = El = Blv$$

Where the upper end is at a higher electric potential than the lower end. **Thus, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.** If the direction of the motion is reversed, the polarity of the potential difference also is reversed.

A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating how a changing magnetic flux causes an induced current in a closed circuit. Consider a circuit consisting of a conducting bar of length sliding along two fixed parallel conducting rails, as shown in Figure 2a.

For simplicity, we assume that the bar has zero resistance and that the stationary part of the circuit has a resistance R. A uniform and constant magnetic field **B** is applied perpendicular to the plane of the circuit. As the bar is pulled to the right with a velocity **v**, under the influence of an applied force $\mathbf{F}_{app}$, free charges in the bar experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. In this case, the rate of change of magnetic flux through the loop and the corresponding induced motional emf across the moving bar are proportional to the change in area of the loop. As we shall see, if the bar is pulled to the right with a constant velocity, the work done by the applied force appears as internal energy in the resistor R.

Because the area enclosed by the circuit at any instant is *lx*, where x is the width of the circuit at any instant, the magnetic flux through that area is

$$\Phi_B = Blx$$

Using Faraday's law, and noting that x changes with time at a rate *dx/dt=v*, we find that the induced motional emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv \qquad (1)$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R} \qquad (2)$$

The equivalent circuit diagram for this example is shown in Figure 2b.

Let us examine the system using energy considerations. Because no battery is in the circuit, we might wonder about the origin of the induced current and the electrical energy in the system. We can understand the source of this current and energy by noting that the applied force does work on the conducting bar, thereby moving charges through a magnetic field. Their movement through the field causes the charges to move along the bar with some average drift velocity, and hence a current is established. Because energy must be conserved, the work done by the applied force on the bar during some time interval must equal the electrical energy supplied by the induced emf during that same interval. Furthermore, if the bar moves with constant speed, the work done on it must equal the energy delivered to the resistor during this time interval.

As it moves through the uniform magnetic field **B**, the bar experiences a magnetic force $\mathbf{F}_B$ of magnitude IlB. The direction of this force is opposite the motion of the bar, to the left in Figure 2a. Because the bar moves with constant velocity, the applied force must be equal in magnitude and opposite in direction to the magnetic force, or to the right in Figure 2a. (If $\mathbf{F}_B$ acted in the direction of motion, it would cause the bar to accelerate. Such a situation would violate the principle of conservation of energy.) Using Equation 2 and the fact that $F_{app} = IlB$, we find that the power delivered by the applied force is

$$p = F_{app}v = (IlB)v = \frac{B^2l^2v^2}{R} = \frac{\varepsilon^2}{R} \qquad (3)$$

We see that this power is equal to the rate at which energy is delivered to the resistor I^2R, as we would expect. It is also equal to the power supplied by the motional emf. This example is a clear demonstration of the conversion of mechanical energy first to electrical energy and finally to internal energy in the resistor.

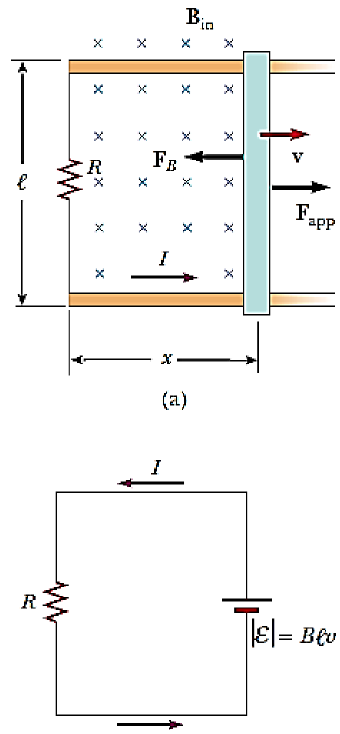


Fig 2: (a) A conducting bar sliding with a velocity v along two conducting rails under the action of an applied force F_{app}. The magnetic force F_B opposes the motion, and a counterclockwise current I is induced in the loop. (b)

Self-Inductance Definition: The amount of induced emf in a solenoid in the response of unit change of current in that same coil is called the self-inductance of that solenoid. Its unit is henry (H).

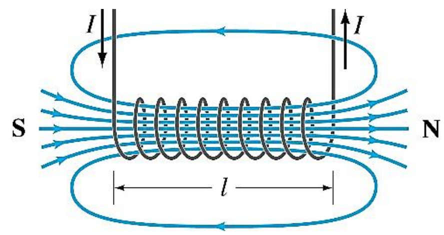
$$L = \frac{\varepsilon}{\frac{dI}{dt}}$$

Mutual Inductance definition: The amount of induced emf in a coil in the response of unit change of current in the coil near to it, is called the mutual inductance of that solenoid. Its unit is henry (H).

$$M = \frac{\varepsilon}{\frac{dI}{dt}}$$

Self-Inductance of a Solenoid

The magnetic flux produced in the inner core of the solenoid is equal to:



$$\Phi = B.A$$

Where: Φ is the magnetic flux, B is the flux density, and A is the area.

If the inner core of a long solenoid coil with N number of turns per meter length is hollow, "air cored", then the magnetic induction within its core will be given as:

$$B = \mu_o H = \mu_o \frac{N.I}{\ell}$$

Then by substituting these expressions in the first equation above for Inductance will give us:

$$L = N\frac{\Phi}{I} = N\frac{B.A}{I} = N\frac{\mu_o.N.I}{\ell.I}.A$$

By cancelling out and grouping together like terms, then the final equation for the coefficient of self-inductance for an air cored coil (solenoid) is given as:

$$L = \mu_o \frac{N^2.A}{\ell}$$

- Where:
- L is in Henries
- μ_o is the Permeability of Free Space ($4.\pi.10^{-7}$ TmA^{-1})
- N is the Number of turns
- A is the Inner Core Area (πr^2) in m^2
- l is the length of the Coil in meters

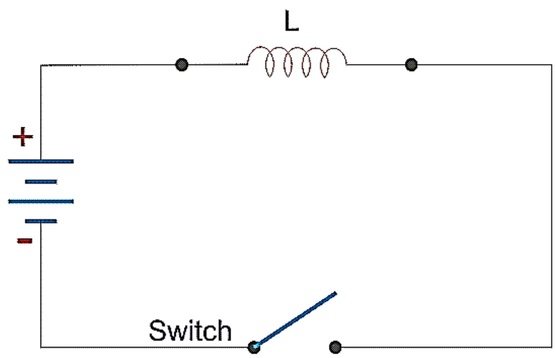
As the inductance of a coil is due to the magnetic flux around it, the stronger the magnetic flux for a given value of current the greater will be the inductance. So a coil of many turns will have a higher inductance value than one of only a few turns and therefore, the equation above will give inductance L as being proportional to the number of turns squared N^2.

Home work: The self-inductance of a coil of 400 turns is 8 mH. What is the magnetic flux through the coil when the current is 5×10^{-8} amp?

Energy stored in a magnetic field

Magnetic field can be of permanent magnet or electro-magnet. Both magnetic fields store some energy. Permanent magnet always creates the magnetic flux and it does not vary upon the other external factors. But electromagnet creates its variable magnetic fields based on how much current it carries. The dimension of this electro-magnet is responsible to create the strength the magnetic field and hence the energy stored in this electromagnet.

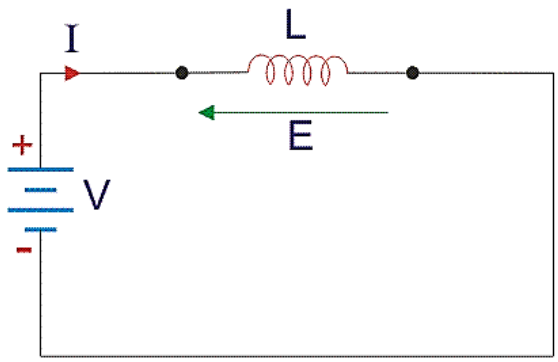
First we consider the magnetic field is due to electromagnet i.e. a coil of several no. turns. This coil or inductor is carrying current I when it is connected across a battery or voltage source through a switch.



Suppose battery voltage is V volts, value of inductor is L Henry, and current I will flow at steady state. When the switch is ON, a current will flow from zero to its steady value. But due to self-induction a induced voltage appears which is

$$E = -L\frac{dI}{dt}$$

This E always in the opposite direction of the rate of change of current.



Now here the energy or work done due to this current passing through this inductor is U. As the current starts from its zero value and flowing against the induced emf E, the energy will grow up gradually from zero value to U.

$$dU = W.dt,$$

Where W is the small power and W = – E.I
So, the energy stored in the inductor is given by

$$dU = W.dt = -E.Idt = L\frac{dI}{dt}.Idt = LIdI$$

Now integrate the energy from 0 to its final value.

$$U = \int_0^U dU = \int_0^I LIdI = \frac{1}{2}LI^2$$

Again,

$$L = \frac{\mu_0 N^2 A}{l}$$

as per dimension of the coil, where N is the number of turns of the coil, A is the effective cross-sectional area of the coil and l is the effective length of the coil.

Again,

$$I = \frac{H.l}{N}$$

Where, H is the magnetizing force, N is the number of turns of the coil and *l* is the effective length of the coil.

$$I = \frac{B.l}{\mu_0.N}$$

Now putting expression of L and I in equation of U, we get new expression i.e.

$$U = \frac{\frac{\mu_0 N^2 A}{l}.\frac{B.l}{\mu_0.N}}{2} = \frac{B^2 Al}{2\mu_0}$$

Ampere’s Law

Oersted’s 1819 discovery about deflected compass needles demonstrates that a current-carrying conductor produces a magnetic field. Figure 1a shows how this effect can be demonstrated in the classroom. Several compass needles are placed in a horizontal plane near a long vertical wire. When no current is present in the wire, all the needles point in the same direction (that of the Earth’s magnetic field), as expected. When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle, as shown in Figure 1b. These observations demonstrate that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule. When the current is reversed, the needles in Figure 1b also reverse.

Because the compass needles point in the direction of **B**, we conclude that the lines of **B** form circles around the wire, as discussed in the preceding section. By symmetry, the magnitude of **B** is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire. By varying the current and distance r from the wire, we find that *B* is proportional to the current and inversely proportional to the distance from the wire.

Now let us evaluate the product **B**.d**s** for a small length element *d***s** on the circular path defined by the compass needles, and sum the products for all elements over the closed circular path. Along this path, the vectors *d***s** and **B** are parallel at each point (see Fig. 1b), so **B**.d**s** = *B ds*. Furthermore, the magnitude of **B** is constant on this circle and is found from Biot-Savart law. Therefore, the sum of the products *B ds* over the closed path, which is equivalent to the line integral of **B**.d**s**, is

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Where $\oint ds = 2\pi r$ is the circumference of the circular path. Although this result was calculated for the special case of a circular path surrounding a wire, it holds for a closed path of any shape surrounding a current that exists in an unbroken circuit.

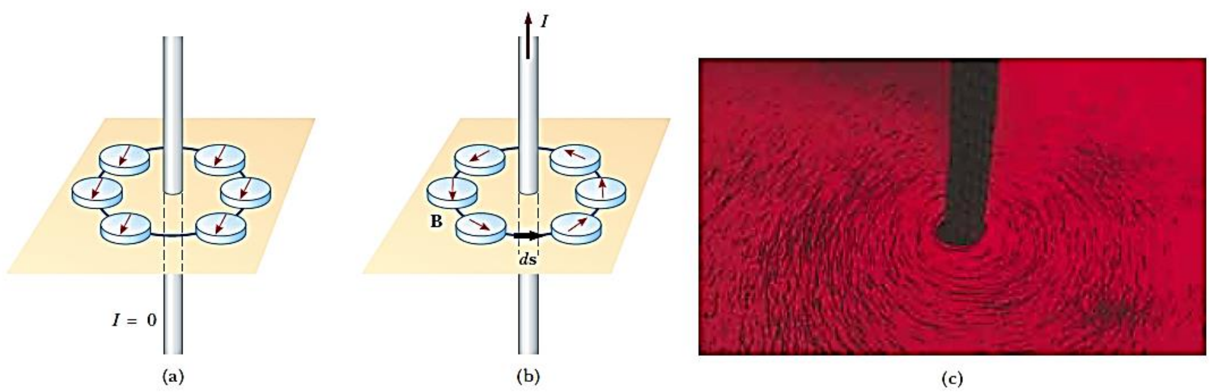


Figure: (a) When no current is present in the wire, all compass needles point in the same direction (toward the Earth's North Pole). (b) When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.

Statement:

The general case, known as Ampère's law, can be stated as follows:

The line integral of **B**.d**s** around any closed path equals $\mu_0 I$, where *I* is the total continuous current passing through any surface bounded by the closed path i.e. the total current passing through the closed loop.

$$\oint \boldsymbol{B}.d\boldsymbol{s} = \mu_0 I$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

Examples

The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire. Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Solution Let us choose for our path of integration circle 1 in Figure 1. From symmetry, **B** must be constant in magnitude and parallel to $d\mathbf{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I_0, Ampère's law gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}} \qquad (\text{for } r \geq R)$$

Note how much easier it is to use Ampère's law than to use the Biot–Savart law.

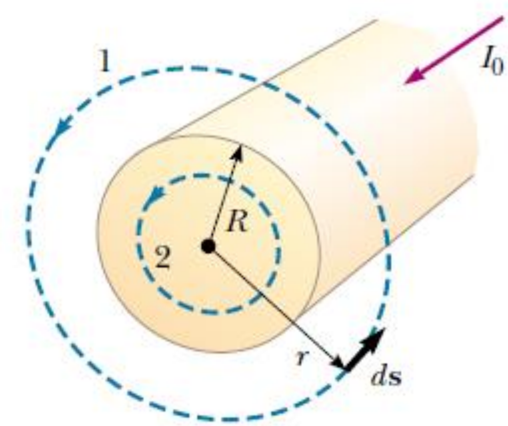


Figure 1 A long, straight wire of radius R carrying a steady current I_0 uniformly distributed across the cross-section of the wire. The magnetic field at any point can be calculated from Ampère's law using a circular path of radius r, concentric with the wire.

This is often the case in highly symmetric situations. Now consider the interior of the wire, where $r < R$. Here the current I passing through the plane of circle 2 is less than the total current I_0 . Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:

$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$I = \frac{r^2}{R^2} I_0$$

Following the same procedure as for circle 1, we apply Ampère's law to circle 2:

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0 \left(\frac{r^2}{R^2} I_0 \right)$$

$$\boxed{B = \left(\frac{\mu_0 I_0}{2\pi R^2} \right) r} \qquad (\text{for } r < R)$$

This result is similar in form to the expression for the electric field inside a uniformly charged sphere. The magnitude of the magnetic field versus r for this configuration is plotted in Figure 2. Note that inside the wire, $B \to 0$ as $r \to 0$. Note also that both equations give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.

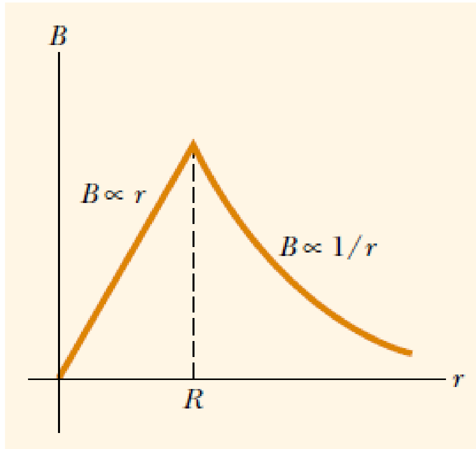


Figure 2 Magnitude of the magnetic field versus r for the wire shown in Figure 30.11. The field is proportional to r inside the wire and varies as $1/r$ outside the wire.

The Magnetic Field Created by a Toroid

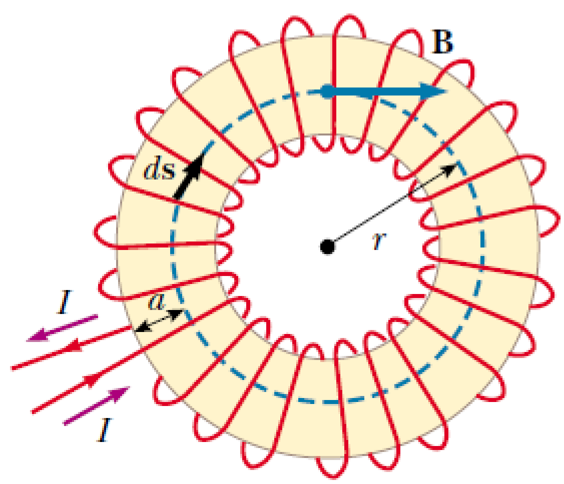
(A device called a toroid (Fig. 2) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a torus) made of a nonconducting material.)

For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the center.

Solution To calculate this field, we must evaluate $\oint \boldsymbol{B}.d\boldsymbol{s}$ over the circle of radius r in Figure 1. By symmetry, we see that the magnitude of the field is constant on this circle and tangent to it, so $\boldsymbol{B}.d\boldsymbol{s} = B\ ds$. Furthermore, note that the circular closed path surrounds N loops of wire, each of which carries a current I. Therefore Ampère's law applied to the circle gives

$$\oint \mathbf{B}\cdot d\mathbf{s} = B\oint ds = B(2\pi r) = \mu_0 NI$$

$$\boxed{B = \frac{\mu_0 NI}{2\pi r}}$$



circle and varies as $1/r$. The field outside the toroid is zero. The dimension a is the cross-sectional radius of the torus.

This result shows that B varies as $1/r$ and hence is nonuniform in the region occupied by the torus. However, if r is very large compared with the cross-sectional radius of the torus, then the field is approximately uniform inside the torus. For an ideal toroid, in which the turns are closely spaced, the external magnetic field is zero. This can be seen by noting that the net current passing through any circular path lying outside the toroid (including the region of the "hole in the doughnut") is zero. Therefore, from Ampère's law we find that $B = 0$ in the regions exterior to the torus.

The Magnetic Field of a Solenoid

Obtain an expression for the interior magnetic field in an ideal solenoid.

(A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the interior of the solenoid—when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns. Figure 1 shows the magnetic field lines surrounding a loosely wound solenoid. Note that the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is uniform and strong. The field lines between current elements on two adjacent turns tend to cancel each other because the field vectors from the two elements are in opposite directions. The field at exterior points such as P is weak because the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion. An ideal solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns. In this case, the external field is zero, and the interior field is uniform over a great volume.)

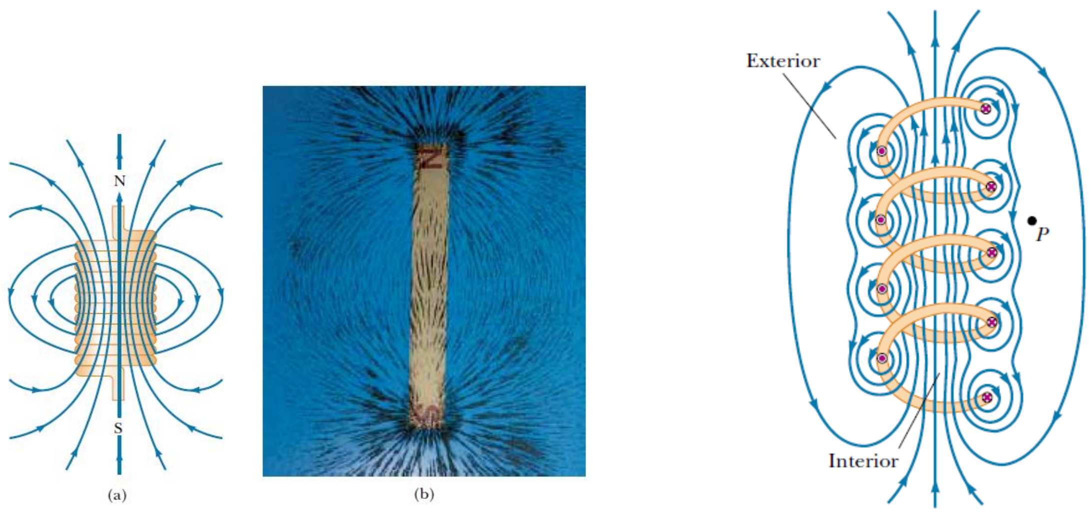


Figure 1 (a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is nearly uniform and strong. Note that the field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.

We can use Ampère's law to obtain an expression for the interior magnetic field in an ideal solenoid. Figure 2 shows a longitudinal cross-section of part of such a solenoid carrying a current *I*. Because the solenoid is ideal, **B** in the interior space is uniform and parallel to the axis, and **B** in the exterior space is zero. Consider the rectangular path of length *l* and width *w* shown in Figure 2. We can apply Ampère's law to this path by evaluating the integral of **B**.d**s** over each side of the rectangle. The contribution along side 3 is zero because B=0 in this region. The contributions from sides 2 and 4 are both zero because **B** is perpendicular to *d***s** along these paths. Side 1 gives a contribution *Bl* to the integral because along this path **B** is uniform and parallel to *d***s**. The integral over the closed rectangular path is therefore

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B}\cdot d\mathbf{s} = B\int_{\text{path 1}} ds = B\ell$$

The right side of Ampère's law involves the total current passing through the area bounded by the path of integration. In this case, the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If N is the number of turns in the length l, the total current through the rectangle is NI. Therefore, Ampère's law applied to this path gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

Where $n = N/l$ is the number of turns per unit length. We also could obtain this result by reconsidering the magnetic field of a toroid. If the radius r of the torus in a toroid containing N turns is much greater than the toroid's cross-sectional radius a, a short section of the toroid approximates a solenoid for which $n = \frac{N}{2\pi r}$.

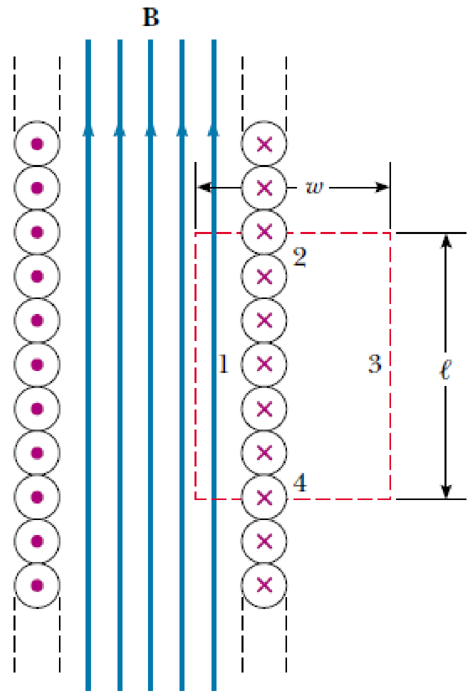


Figure 2 Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is zero. Ampère's law applied to the red dashed path can be used to calculate the magnitude of the interior field

Galvanometers

A galvanometer is a device that is used to detect small electric current or measure its magnitude. The current and its intensity is usually indicated by a magnetic needle's movement or that of a coil in a magnetic field that is an important part of a galvanometer.

Since its discovery in the 1800s, galvanometer has seen many iterations. Some of the different types of galvanometer include Tangent galvanometer, Astatic galvanometer, Mirror galvanometer and Ballistic galvanometer. However, today the main type of galvanometer type that is used widely is the D'Arsonval/Weston type or the moving coil type. A galvanometer is basically a historical name that has been given to a moving coil electric current detector.

Moving Coil Galvanometer

What is a Moving Coil Galvanometer?

A moving coil galvanometer is an instrument which is used to measure electric currents. It is a sensitive electromagnetic device which can measure low currents even of the order of a few microamperes.

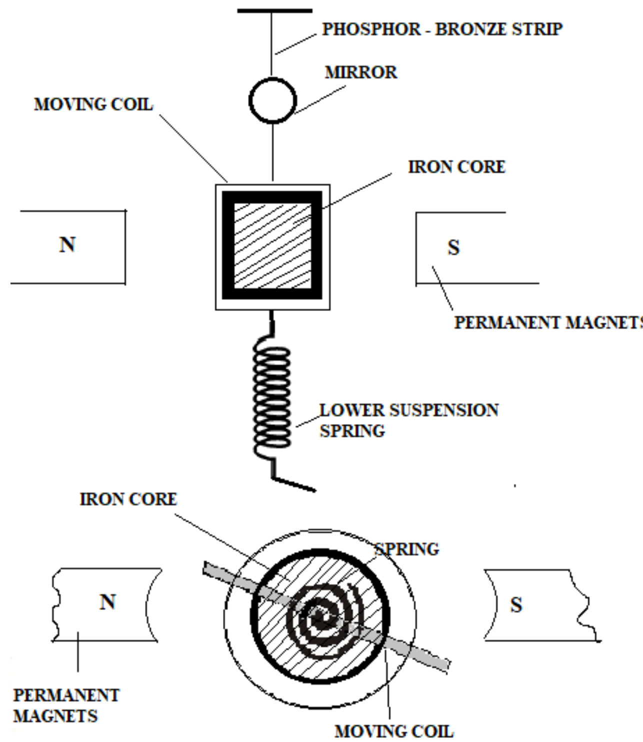
Moving-coil galvanometers are mainly divided into two types:

- Suspended coil galvanometer
- Pivoted-coil or Weston galvanometer

Moving Coil Galvanometer Principle

A current-carrying coil when placed in an external magnetic field experiences magnetic torque. The angle through which the coil is deflected due to the effect of the magnetic torque is proportional to the magnitude of current in the coil.

Moving Coil Galvanometer Construction and Diagram

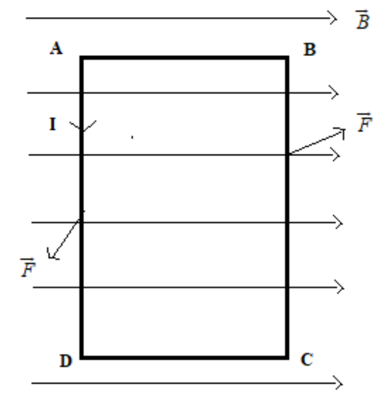


The moving coil galvanometer is made up of a rectangular coil that has many turns and it is usually made of thinly insulated or fine copper wire that is wounded on a metallic frame. The coil is free to rotate about a fixed axis. A phosphor-bronze strip that is connected to a movable torsion head is used to suspend the coil in a uniform radial magnetic field.

Essential properties of the material used for suspension of the coil are conductivity and a low value of the torsional constant. A cylindrical soft iron core is symmetrically positioned inside the coil to improve the strength of the magnetic field and to make the field radial. The lower part of the coil is attached to a phosphor-bronze spring having a small number of turns. The other end of the spring is connected to binding screws.

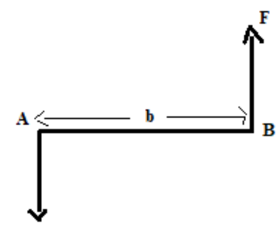
The spring is used to produce a counter torque which balances the magnetic torque and hence help in producing a steady angular deflection. A plane mirror which is attached to the suspension wire, along with a lamp and scale arrangement is used to measure the deflection of the coil. Zero-point of the scale is at the center.

Working of Moving Coil Galvanometer



Let a current I flow through the rectangular coil of n number of turns and a cross-sectional area A. When this coil is placed in a uniform radial magnetic field B, the coil experiences a torque τ.

Let us first consider a single turn ABCD of the rectangular coil having a length *l* and breadth *b***.** This is suspended in a magnetic field of strength B such that the plane of the coil is parallel to the magnetic field. Since the sides AB and DC are parallel to the direction of the magnetic field, they do not experience any effective force due to the magnetic field. The sides AD and BC being perpendicular to the direction of field experience an effective force F given by F = I*l*B



Using Fleming's left-hand rule we can determine that the forces on AD and BC are in opposite direction to each other. When equal and opposite forces F called couple acts on the coil, it produces a torque. This torque causes the coil to deflect.

We know that torque τ = force x perpendicular distance between the forces

$$\boldsymbol{\tau} = \mathbf{F} \times \boldsymbol{b}$$

Substituting the value of F we already know,

Torque acting on single-loop ABCD of the coil, $\boldsymbol{\tau} = BI\boldsymbol{l} \times \boldsymbol{b}$

Where ***l*x *b*** is the area A of the coil,

Hence the torque acting on n turns of the coil is given by

$$\tau = nIAB$$

The magnetic torque thus produced causes the coil to rotate, and the phosphor bronze strip twists. In turn, the spring S attached to the coil produces a counter torque or restoring torque kθ which results in a steady angular deflection.

Under equilibrium condition:

$$C\theta = nIAB$$

Here C is called the torsional constant of the spring (restoring couple per unit twist). The deflection or twist θ is measured as the value indicated on a scale by a pointer which is connected to the suspension wire.

$$\theta = (nAB / C)I$$

Therefore $\theta \propto I$

$$\theta = k I$$

The quantity $k = nAB / C$ is a constant for a given galvanometer. Hence it is understood that the deflection that occurs the galvanometer is directly proportional to the current that flows through it.

Sensitivity of Moving Coil Galvanometer

The general definition of the sensitivity experienced by a moving coil galvanometer is given as the ratio of change in deflection of the galvanometer to the change in current in the coil.

$$S = d\theta/dI$$

The sensitivity of a galvanometer is higher if the instrument shows larger deflection for a small value of current. Sensitivity is of two types, namely current sensitivity and voltage sensitivity.

Current Sensitivity

The deflection θ per unit current I is known as current sensitivity θ/I

$$\theta/I = nAB/C = k$$

Voltage Sensitivity

The deflection θ per unit voltage is known as Voltage sensitivity θ/V. Dividing both sides by V in the equation $\theta = (nAB / C)I$;

$$\theta/V = (nAB /V k)I = (nAB / k)(I/V) = (nAB /k)(1/R)$$

R stands for the effective resistance in the circuit.

It is worth noting that voltage sensitivity = Current sensitivity/ Resistance of the coil. Therefore under the condition that R remains constant; voltage sensitivity $\propto$ Current sensitivity.

Figure of Merit of a Galvanometer

It is the ratio of the full-scale deflection current and the number of graduations on the scale of the instrument. It also the reciprocal of the current sensitivity of a galvanometer.

Factors Affecting Sensitivity Of A Galvanometer

a) Number of turns in the coil

b) Area of the coil

c) Magnetic field strength B

d) The magnitude of couple per unit twist k/nAB

Solved Question: Increase in current sensitivity results in an increase in voltage sensitivity of a moving coil galvanometer. Yes or no? Justify your answer.

Solution: No. An increase in current sensitivity of a moving coil galvanometer may not necessarily result in an increase in voltage sensitivity. As the number of turns (length of the coil) are increased to increase the current sensitivity of the device, the resistance of the coil changes. This is because the resistance of the coil is dependent on factors like the length and area of the coil.

As we know that voltage sensitivity $\theta/V = (nAB/k)(1/R)$; the overall value of voltage sensitivity remains unchanged.

Applications of Galvanometer

The moving coil galvanometer is a highly sensitive instrument due to which it can be used to detect the presence of current in any given circuit. If a galvanometer is a connected in a Wheatstone's bridge circuit, pointer in the galvanometer shows null deflection, i.e no current flows through the device. The pointer deflects to the left or right depending on the direction of the current.

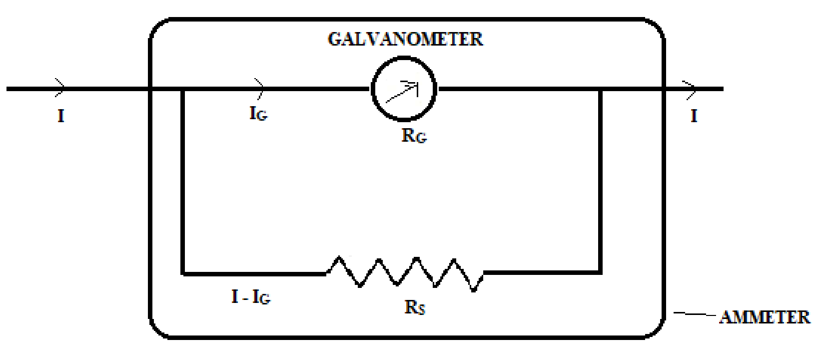
The galvanometer can be used to measure;

a) the value of current in the circuit by connecting it in parallel to low resistance.

b) the voltage by connecting it in series with high resistance.

Conversion of Galvanometer to Ammeter

A galvanometer is converted into an ammeter by connecting it in parallel with a low resistance called shunt resistance. Suitable shunt resistance is chosen depending on the range of the ammeter.



CONVERSION OF GALVANOMETER INTO AMMETER

In the given circuit

R_G – Resistance of the galvanometer

G- Galvanometer coil

I – Total current passing through the circuit

I_G – Total current passing through the galvanometer which corresponds to full-scale reading

R_s – Value of shunt resistance

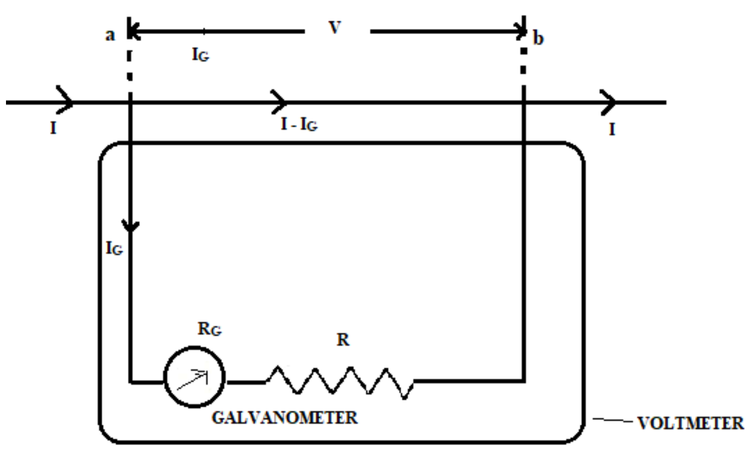
When current I_G passes through the galvanometer, the current through the shunt resistance is given by $I_S = I – I_G$. The voltages across the galvanometer and shunt resistance are equal due to the parallel nature of their connection.

Therefore $R_G .I_G= (I- I_G).R_s$

The value of S can be obtained using the above equation.

Conversion of Galvanometer to Voltmeter

A galvanometer is converted into an ammeter by connecting it in series with high resistance. A suitable high resistance is chosen depending on the range of the ammeter.



CONVERSION OF GALVANOMETER INTO VOLTMETER

In the given circuit

R_G = Resistance of the galvanometer

R = Value of high resistance

G = Galvanometer coil

I = Total current passing through the circuit

I_G = Total current passing through the galvanometer which corresponds to a full-scale deflection.

V = Voltage drop across the series connection of galvanometer and high resistance.

When current I_G passes through the series combination of the galvanometer and the high resistance R; the voltage drop across the branch ab is given by

$V = R_G.I_G + R.I_G$

The value of R can be obtained using the above equation.

Solved Question: A moving coil galvanometer of resistance 100Ω is used as an ammeter using a resistance of 0.1Ω. The maximum deflection current in the galvanometer is 100µA. Find the current in the circuit, so that the ammeter shows maximum deflection.

Solution: It is given that $R_G = 100\Omega$, $R_s = 0.1\Omega$, $I_G = 100\mu A$

We know that $R_G .I_G = (I - I_G).R_S$

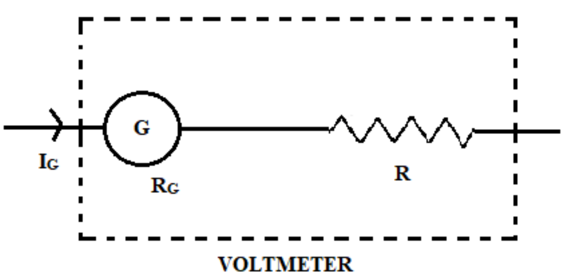
Therefore $I = (R_G .I_G + I_G.R_s)/ R_S$

$I = (1 + R_G/ R_S). I_G$

Substituting the given values, we get I= 100.1mA

Solved Question: A galvanometer coil of 40Ω resistance shows full range deflection for a current of 4mA. How can this galvanometer be converted into a voltmeter of range 0-12V?

Solution:



As we know that $V = I_G (R_G + R)$

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$R = V/ I_G - R_G$

$= (12/ (4\times10^{-3})) - 40$

$R = 2960\ \Omega$

Advantages and Disadvantages of a Moving Coil Galvanometer

Advantages

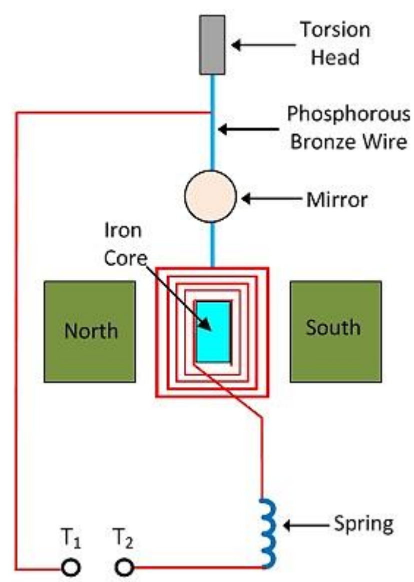
- High sensitivity.
- Not easily affected by stray magnetic fields.
- The torque to weight ratio is high.
- High accuracy and reliability.

Disadvantages

- It can be used only to measure direct currents.
- Develops errors due to factors like aging of the instrument, permanent magnets and damage of spring due to mechanical stress.

Ballistic Galvanometers

The galvanometer which is used for estimating the quantity of charge flow through it is called the ballistic galvanometer. The working principle of the ballistic galvanometer is very simple. It depends on the deflection of the coil which is directly proportional to the charge passes through it. The galvanometer measures the majority of the charge passes through it in spite of current.



Ballistic Galvanometer

Construction of Ballistic Galvanometer

The ballistic galvanometer consists coil of copper wire which is wound on the non-conducting frame of the galvanometer. The phosphorous bronze suspends the coil between the north and south poles of a magnet. For increasing the magnetic flux the iron core places within the coil. The lower portion of the coil connects with the spring. This spring provides the restoring torque to the coil.

When the charge passes through the galvanometer, their coil starts moving and gets an impulse. The impulse of the coil is proportional to the charges passes through it. The actual reading of the galvanometer achieves by using the coil having a high moment of inertia. The moment of inertia means the body oppose the angular movement. If the coil has a high moment of inertia, then their oscillations are large. Thus, accurate reading is obtained.

Theory of Ballistic Galvanometer

Consider the rectangular coil having N number of turns placed in a uniform magnetic field. Let **l** be the length and **b** be the breadth of the coil. The area of the coil is given as

$$A = l \times b \ldots..equ(1)$$

When the current passes through the coil, the torque acts on it. The given expression determines the magnitude of the torque.

$$\tau = NiBA \ldots \ldots equ(2)$$

Let the current flow through the coil for very short duration says dt and it is expressed as

$$\tau dt = NiBAdt \ldots \ldots equ(3)$$

If the current passing through the coil for t seconds, the expression becomes

$$\int_0^t \tau dt = NBA \int_0^t idt = NBAq \ldots \ldots equ(4)$$

The q be the total charge passes through the coil. The moment of inertia of the coil is given by **l,** and the angular velocity through **ω.** The expression gives the angular momentum of the coil

$$Angular\ momentum\ =\ l\omega \ldots \ldots.equ(5)$$

The angular momentum of the coil is equal to the force acting on the coil. Thus from equation (4) and (5), we get.

$$l\omega = NBAq \ldots \ldots equ(6)$$

The Kinetic Energy (K) deflects the coil through an angle θ, and this deflection is restored through the spring.

$$\text{Restoring torque} = \frac{1}{2}c\theta^2$$

$$\text{Kinetic energy K} = \frac{1}{2}l\omega^2$$

The resorting torque of the coil is equal to their deflection. Thus,

$$\frac{1}{2}c\theta^2 = \frac{1}{2}l\omega^2$$

$$c\theta^2 = l\omega^2 \ldots\ldots\ldots.equ(7)$$

The periodic oscillation of the coil is given as

$$T = 2\pi\sqrt{l/_c}$$

$$T^2 = \frac{4\pi^2 l}{c}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{c}$$

$$\frac{cT^2}{4\pi^2} = l$$

By multiplying the equation (7) from the above equation we get

$$\frac{c^2T^2\theta^2}{4\pi^2} = l^2\omega^2$$

$$\frac{ct\theta}{2\pi} = l\omega \ldots\ldots\ldots.equ(8)$$

On substituting the value of equation (6) in the equation (8) we get

$$\frac{ct\theta}{2\pi} = NBAq$$

$$q = \frac{ct\theta}{NBA2\pi} \dots\dots\dots.\, equ(9)$$

$$q = \frac{ct}{2\pi BNA} \times (\theta)$$

$$Let, k = \frac{ct}{2\pi BNA}$$

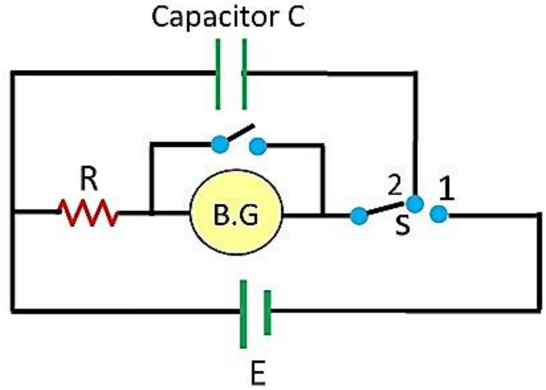
$$q = k\theta$$

The K is the constant of the ballistic galvanometer.

Calibration of Galvanometer Using a Capacitor

The calibration of the galvanometer is the process of determining its constant value by the help of the practical experiments. The following are the methods used for determining the constant of the ballistic galvanometer.

The charging and discharging of the capacitor gives the values of the ballistic galvanometer constant. The circuit arrangement for the calibration of a ballistic galvanometer using the capacitor is shown in the figure below.



Calibration of Ballistic Galvanometer

The circuit uses two pole switch S and the unknown EMF source E. When the switch S connects to terminal 2 then the capacitor becomes charged. Similarly, when the switch connects to terminal 1, then the capacitor becomes discharges through the resistor R, connected in series with the ballistic galvanometer.

The discharge current of the capacitor deflects the coil of ballistic galvanometer through an angle θ. The formula calculates constant of the galvanometer

$$K_q = {}^{Q}/_{\theta_1} = {}^{CE}/_{\theta_1}\, columb/radian$$

Deadbeat Galvanometers

When current is passed through a galvanometer, the coil oscillates about its mean position before coming to rest. To bring the coil to rest immediately, the coil is wound on a metallic frame. Now, when the coil oscillates, eddy currents are set up in a metallic frame, which opposes further oscillations of the coil. This in turn enables the coil to attain its equilibrium position almost instantly. Since the oscillations of the coil die out instantaneously, the galvanometer is called dead beat galvanometer.

Ballistic and dead beat galvanometer both are normal moving coil galvanometers only, In Ballistic galvanometer the coil has higher moment on Inertia and finds its use in measurement of **Charge,** while on the other hand the Dead Beat Galvanometer is simple moving coil galvanometer which employs **eddy current damping** as the damping mechanism thus making it reach steady state value faster

References:

- **Physics (5th edition) – Halliday, Resnick, Krane**
- **https://byjus.com/jee/galvanometer/**
- **https://circuitglobe.com/**

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/ballistic-galvanometer-equation-555555.jpg

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Calibration of Galvanometer

7/10/ballistic-galvanometer-equation-of-charge.jpg **meter is the process of determining its**

Calibration of Galvanometer

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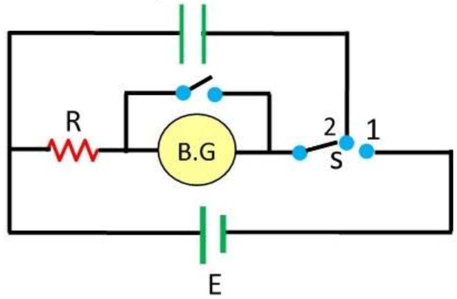

Calibration of Ballistic Galvanometer with Capacitor

Circuit Globe

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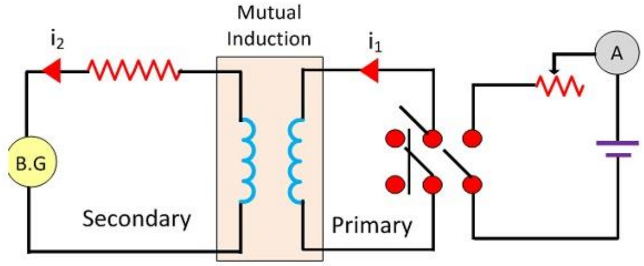
Using a Mutual Inductance

The constant of the ballistic galvanometer determines through the mutual inductance between the coils. The arrangement of the ballistic galvanometer requires two coils; primary and secondary. The primary coil is energised by knowing voltage source.



Calibration of Ballistic Galvanometer by mutual Induction

Circuit Globe

Because of the mutual induction, the current induces in the secondary circuit.