Online Lecture 2

EEE4231:Control system Section: A

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Contents:

- Objective of todays lecture.
- Differential equation and transfer function.
- Block diagram and transfer function
- Transfer function of electrical system.
- Learning outcomes.

Objectives of todays lecture

- To understand transfer function
- To write transfer function from differential equation and vice versa.
- Exercise and home work.
- Writing transfer function from block diagram.
- To write transfer function of an electrical system.
- Exercise and home work

Transfer function

Transfer function is representation of input and output relationship in terms of s domain where s is a complex variable. The transfer function can be obtained by inspection or by simple algebraic manipulations of the differential equations that describe the systems.

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t) \qquad (2.50)$$

where $c(t)$ is the output, $r(t)$ is the input, and the a_i's, b_i's, and the form of the differential equation represent the system. Taking the Laplace transform of both sides,

$$\begin{aligned} & a_n s^n C(s) + a_{n-1} s^{n-1} C(s) + \cdots + a_0 C(s) + \text{initial condition} \\ & \qquad \text{terms involving } c(t) \\ & = b_m s^m R(s) + b_{m-1} s^{m-1} R(s) + \cdots + b_0 R(s) + \text{initial condition} \\ & \qquad \text{terms involving } r(t) \end{aligned} \qquad (2.51)$$

Equation (2.51) is a purely algebraic expression. If we assume that *all initial conditions are zero*, Eq. (2.51) reduces to

$$(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)C(s) = (b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)R(s) \tag{2.52}$$

Now form the ratio of the output transform, $C(s)$, divided by the input transform, $R(s)$:

$$\boxed{\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}} \tag{2.53}$$

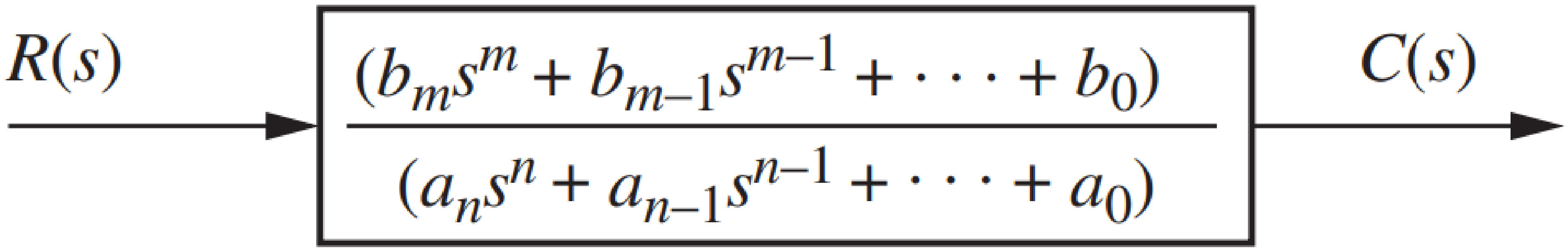


FIGURE 2.2 Block diagram of a transfer function

Lets solve it for various input functions:

Example 2.4

Transfer Function for a Differential Equation

PROBLEM: Find the transfer function represented by

$$\frac{dc(t)}{dt} + 2c(t) = r(t) \qquad (2.55)$$

1. Step input $r(t) = u(t) = \begin{cases} 1; t \geq 0 \\ 0; t < 0 \end{cases}$

Soln:

sC(s)+2C(s)=R(s)/s , C(s)/R(s)=G(s)=1/s(s+2) which is the transfer function.

For time domain response we must use inverse Laplace transform then the transform function can be converted into an standard form using the method explained in (page ,) where it is applicable to follow Case 1(page 37-39).

Exercise (continued)

G(s) can be written using partial expansion

$$\frac{1}{s(s+2)} = \frac{k1}{s} + \frac{k2}{s+2}$$

Now to find the value of k1 and k2 (follow the method case 1, page 37-39)

K1=1/2 and k2=-1/2

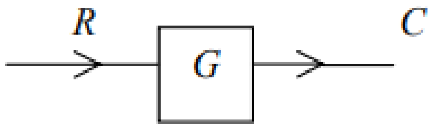
Therefore the time domain solution is

$c(t) = \frac{1}{2} - 1/2e^{-2t}$ which is the time domain response of the given system represented by the Eq. (2.55).

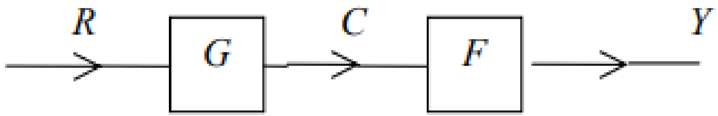
2. Ramp input (assignment)

4. Write differential equation in time domain from the following transfer function: $G(s) = \frac{1}{s(s+2)(s+4)}$

Transfer function block diagram

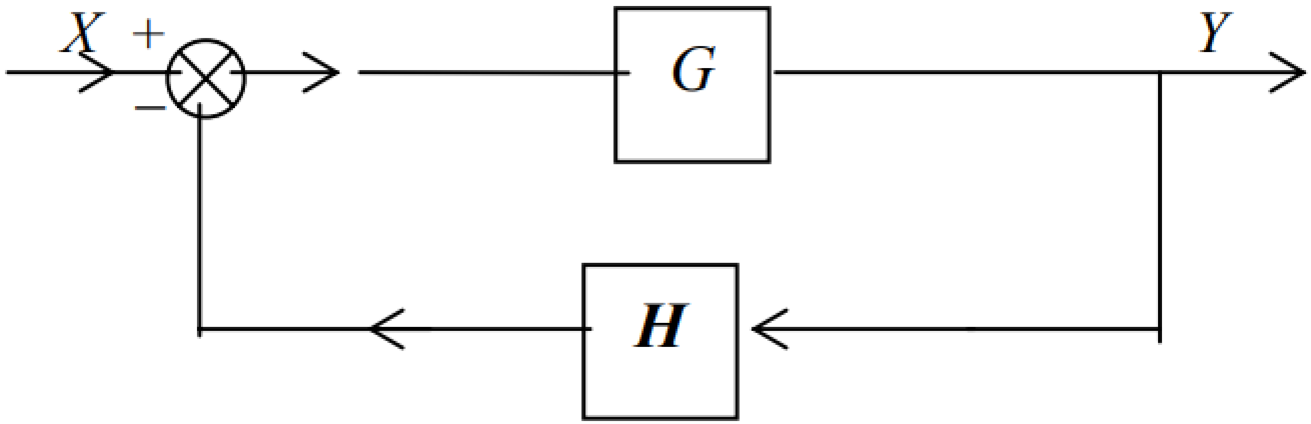


$$C = GR$$



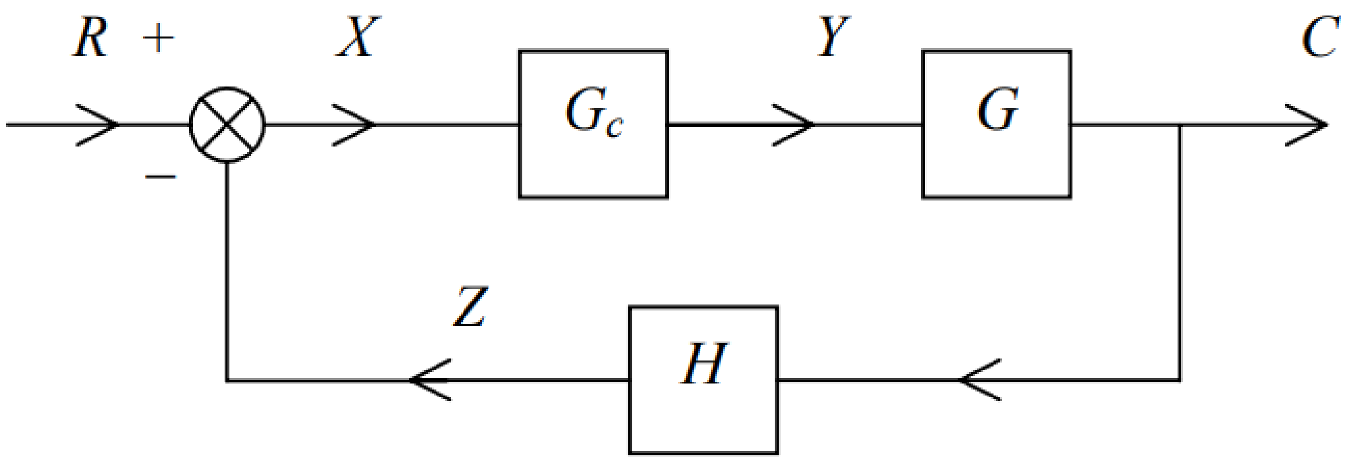
$$C = GR$$
$$Y = FC$$

$$Y = FGR$$



Closed loop Transfer function

$$T = \frac{Y}{X} = \frac{G}{1+GH}$$



$$X = R\text{-}Z$$
$$Y = G_c X$$
$$C = GY$$

and $Z = HC$ (known as feedback)

$$X = R - GG_cHX$$

that is $$X = \frac{R}{1 + GG_cH}$$

and $$\frac{C}{R} = \frac{G_cG}{1 + GG_cH} = T$$

Name	Time domain Voltage, v_L	Transfer function	Impedance Z(s)
inductor	$v_L = L\frac{di}{dt}$	$Ls\, I(s)$	Ls
Capacitor	$v_c = \frac{1}{c}\int idt$	$\frac{1}{cs} I(s)$	$\frac{1}{cs} I$
Resistor	iR	$I(s)\, R$	R

Examples: Transfer function of an RLC circuit

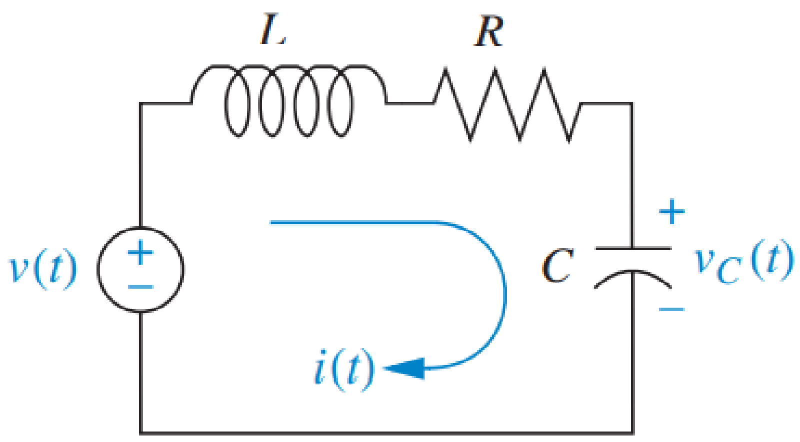


FIGURE 2.3 *RLC* network

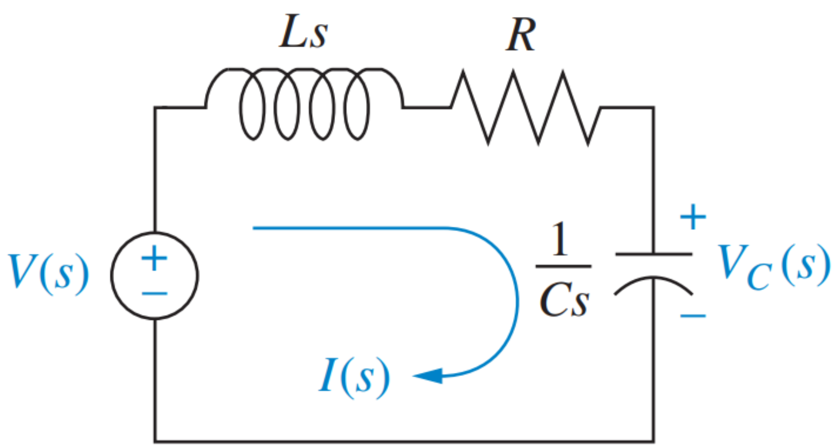


FIGURE 2.5 Laplace-transformed network

$$\left\{Ls + R + \frac{1}{Cs}\right\} \mathrm{I(s)} = \mathrm{V(s)}$$

$$V_c(s) = V(s) - I(s)\{Ls + R\}$$

Find G(s) from these two equations.

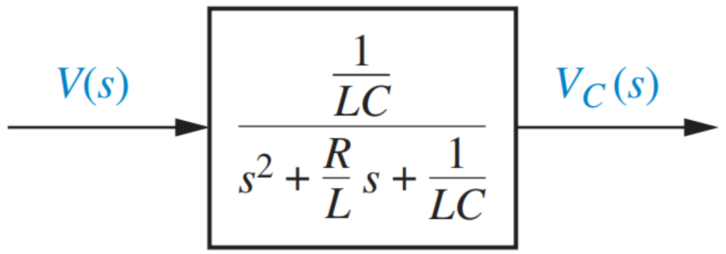


FIGURE 2.4 Block diagram of series *RLC* electrical network

Learning outcomes:

- Able to define transfer function
- Able to write transfer function from differential equation.
- Able to solve examples.
- Able to derive transfer function from block diagram.
- Able to write transfer function of basic electrical components.

END and complete home work