

EXCAVATION DESIGN IN MASSIVE ROCK

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Ref. Book
Rock Mechanics for Underground Mining (3rd Edition)
B.H.G. Brady & E.T. Brown

Chapter-7

General principles of excavation design

Mining excavations are basically of two types- service openings and production openings.

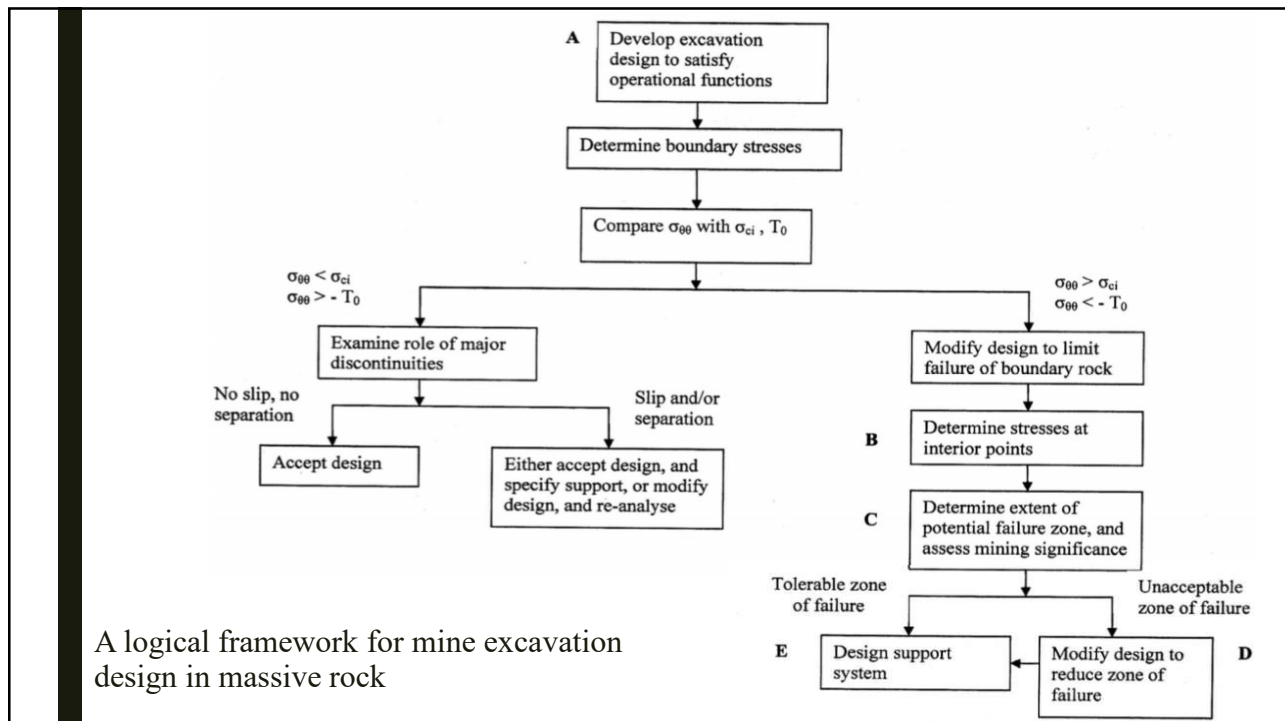
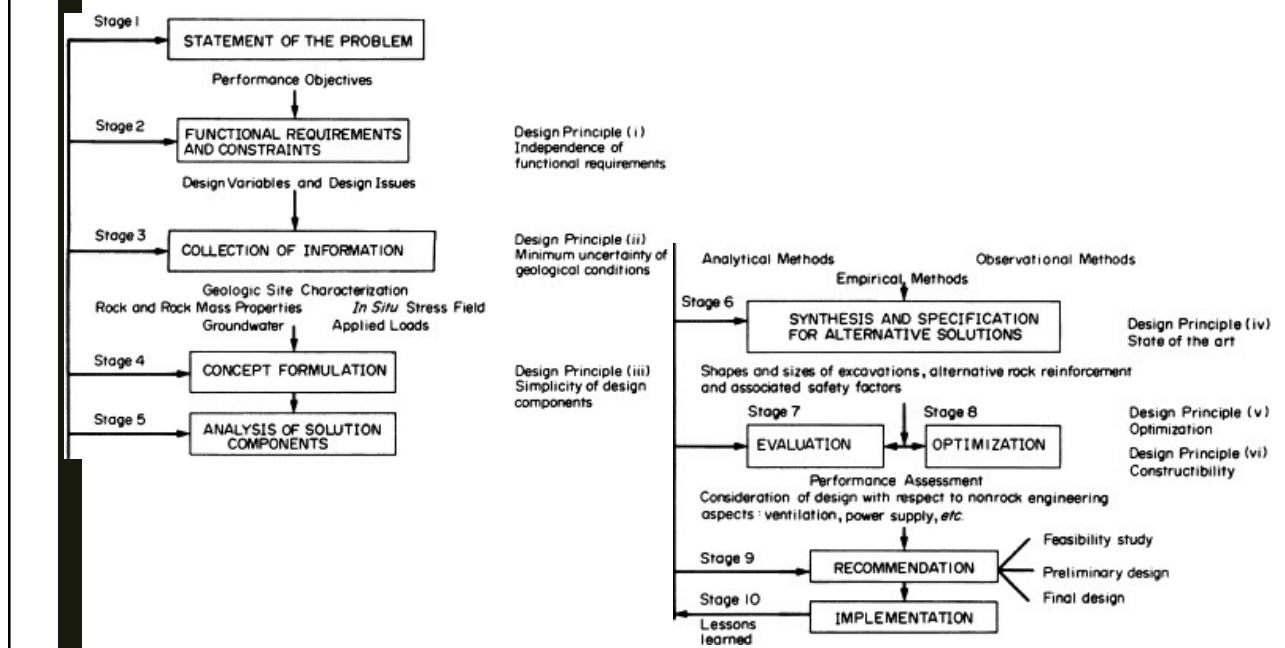
Service openings – Service openings include mine access, ore haulage drive, airway, crusher chambers and underground workshop space.

They are characterized by a duty life approaching the mining life of the orebody, need to assure and maintain at low cost over a relatively long operational life.

Production openings –These openings include ore sources, stopes, and related excavations such as drill headings, stope access and ore extraction and service ways.

Mine production openings have a temporary function in operation and is necessary to assure control of around excavation boundary only for the life of stope (as short as a few months).

A general design methodology for rock engineering (after Bieniawski, 1993)



Zone of Influence

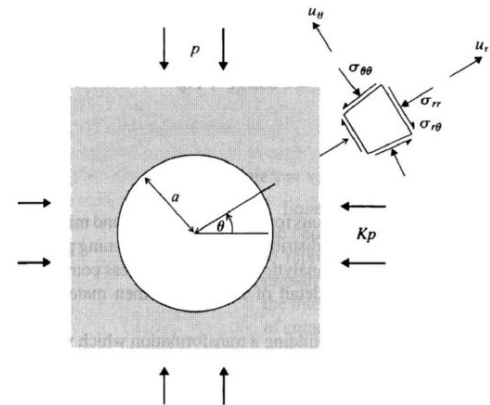
The zone of influence is a domain of significant disturbance of the pre-mining stress field by an excavation. Depends on excavation shape and pre-mining stresses. Stress distribution around a circular openings (Kirsch equations).

General case:-

$$\sigma_{rr} = \frac{p}{2} \left[(1+K) \left(1 - \frac{a^2}{r^2} \right) - (1-K) \left(1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos 2\theta \right]$$

$$\sigma_{\theta\theta} = \frac{p}{2} \left[(1+K) \left(1 + \frac{a^2}{r^2} \right) + (1-K) \left(1 + 3 \frac{a^4}{r^4} \right) \cos 2\theta \right]$$

$$\sigma_{r\theta} = \frac{p}{2} (1-K) \left(1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin 2\theta$$



Hydrostatic stress case:-

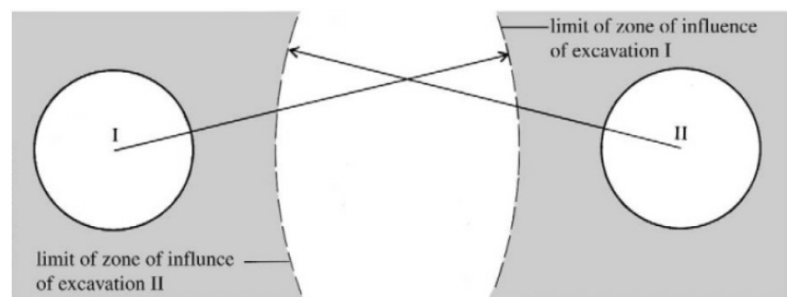
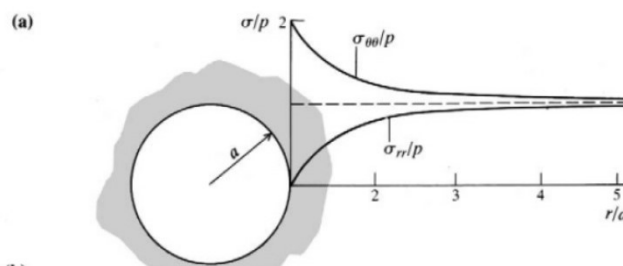
$$\sigma_{rr} = p \left(1 - \frac{a^2}{r^2} \right)$$

$$\sigma_{\theta\theta} = p \left(1 + \frac{a^2}{r^2} \right)$$

$$\sigma_{r\theta} = 0$$

Openings of the same radius

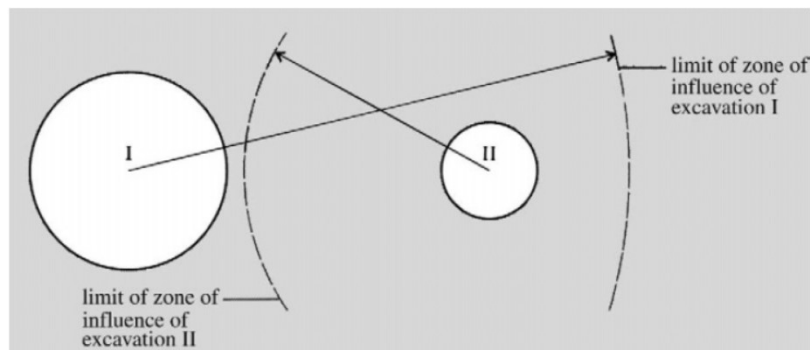
$D_{I,II} \geq 6a$ (within $\pm 5\%$ from the field stresses).



Openings of the different radius

General rule: openings lying outside one another's zones of influence can be designed by ignoring the presence of all others.

Boundary stresses around II can be obtained by calculating the state of stress at the center of II which is adopted as the far-field stresses in the Kirsch equations, prior to its excavation.



Elliptic opening

General shapes of openings can be represented by ellipses inscribed in the opening cross sections. Zone of influence of an elliptic excavation-

$$W_1 = H[A\alpha |q(q+2) - K(3+2q)]^{1/2}$$

Or

$$W_1 = H[\alpha \{A(K+q^2) + Kq^2\}]^{1/2}$$

$$H_1 = H[A\alpha |K(1+2q) - q(3q+2)]^{1/2}$$

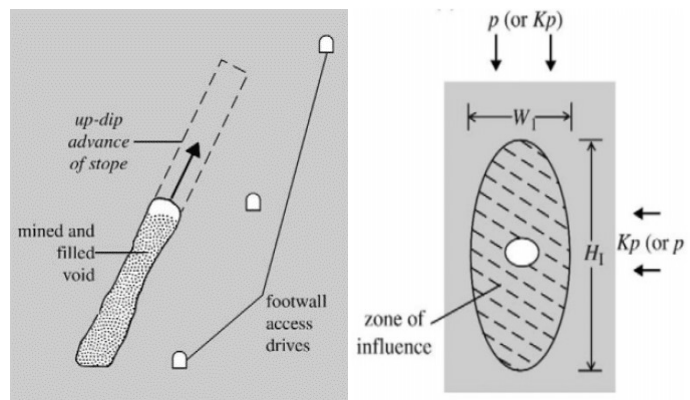
Or

$$H_1 = H[\alpha \{A(K+q^2) + 1\}]^{1/2}$$

where W and H are width and height of the elliptical excavation,

$q = W/H$, $A = 100/2c$ and $\alpha = 1$,

If $K < 1$, and $\alpha = 1/K$, if $K > 1$



Effect of planes of weakness on elastic stress distribution

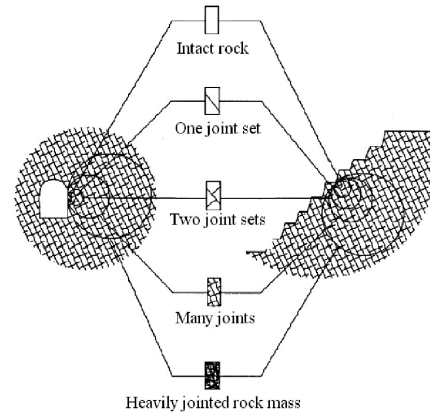
Elastic analysis for the excavations with discontinuities

- In some cases, provides a perfectly valid basis for design
- or a basis for judgment of engineering significance of a discontinuity.

Basic assumption of discontinuities

- Zero tensile strength
- Non-dilatant in shear
- Shear strength follows

$$\tau = \sigma_n \tan \phi$$



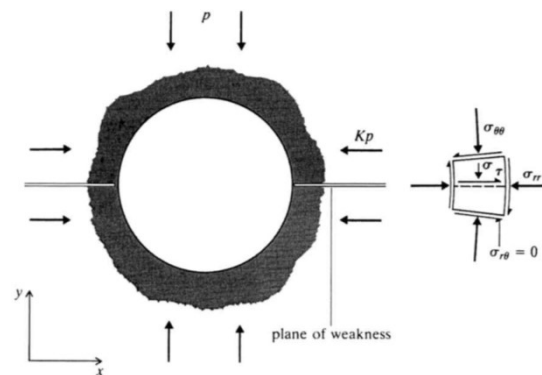
Case 1: Horizontal discontinuity passing through the opening center

- $\sigma_{r\theta} = 0$ for all r at $\theta = 0$: no slip, σ_{rr} and $\sigma_{\theta\theta}$ are principal stresses.
- The plane of weakness (discontinuity) has no effect on the elastic stress distribution.

$$\sigma_{rr} = \frac{p}{2} \left[(1+K) \left(1 - \frac{a^2}{r^2} \right) - (1-K) \left(1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos 2\theta \right]$$

$$\sigma_{\theta\theta} = \frac{p}{2} \left[(1+K) \left(1 + \frac{a^2}{r^2} \right) + (1-K) \left(1 + 3 \frac{a^4}{r^4} \right) \cos 2\theta \right]$$

$$\sigma_{r\theta} = \frac{p}{2} (1-K) \left(1 + 2 \frac{a^2}{r^2} - 3 \frac{a^4}{r^4} \right) \sin 2\theta$$



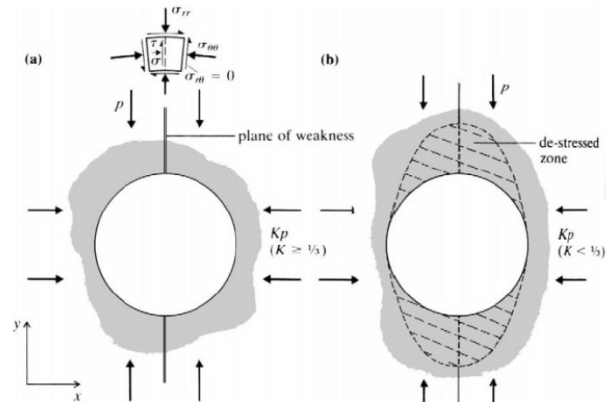
Case 2: Vertical discontinuity passing through the opening center

- The possibility of separation on the plane of weakness arises if tensile stress can develop in the crown of the opening, i.e. if $K < 1/3$.
- If $K \geq 1/3$, the elastic stress distribution is unaltered by either slip or separation. $\sigma_{r\theta} = 0$ for all r at $\theta = 90$: no slip, σ_{rr} and $\sigma_{\theta\theta}$ are principal stresses.

$$\sigma_B = p \left(K - 1 + \frac{2K}{q} \right) = 0$$

$$\text{or } q = \frac{2K}{1-K} \quad \left(q = \frac{W}{H} = \frac{2a}{H} \right)$$

$$\Delta h = a \left(\frac{1-3K}{2K} \right)$$



Case 3: Horizontal discontinuity passing through the opening

- Normal & shear stresses at the intersections on boundary

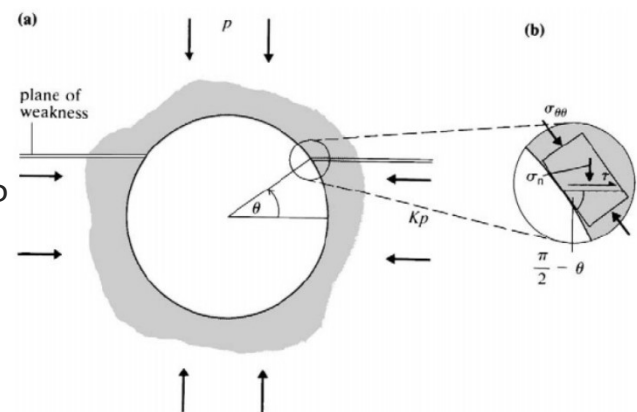
$$\sigma_n = \sigma_{\theta\theta} \cos^2 \theta$$

$$\tau = \sigma_{\theta\theta} \sin \theta \cos \theta$$

$$= \sigma_n \tan \phi \text{ (slip occurs)}$$

$$\rightarrow \sigma_{\theta\theta} \sin \theta \cos \theta = \sigma_{\theta\theta} \cos^2 \theta \tan \phi$$

$$\tan \theta = \tan \phi \text{ or } \sigma_{\theta\theta} \frac{\sin(\theta - \phi)}{\cos \phi} = 0$$



- Slip occurs when $\theta = \phi$ or $\sigma_{\theta\theta} = 0$
- Intersection regions are either de-stressed or at confining stress.

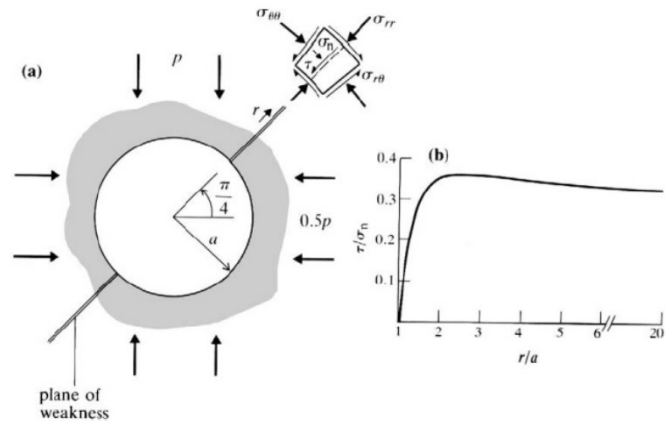
Case 4: Arbitrary discontinuity passing through the opening center

- Normal & shear stresses on the weak plane

$$\sigma_n = \sigma_{\theta\theta} = \frac{p}{2} \left(1.5 \left(1 + \frac{a^2}{r^2} \right) \right)$$

$$\tau = \sigma_{r\theta} = \frac{p}{2} \left(0.5 \left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \right)$$

- Slip does not occur if $\phi > 19.6^\circ$



Case 5: Horizontal discontinuity not intersecting the opening

- Normal & shear stresses on the weak plane

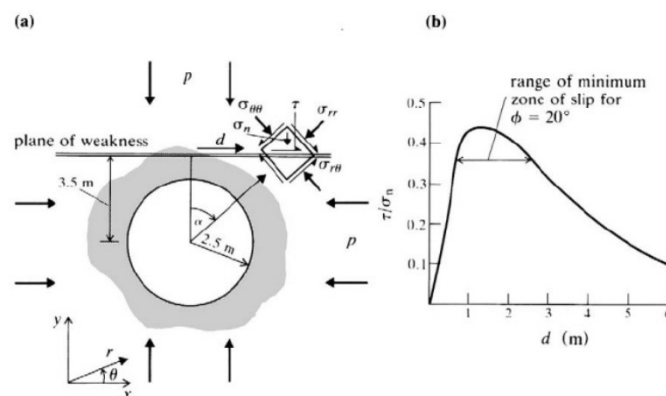
$$\sigma_n = \frac{1}{2}(\sigma_{rr} + \sigma_{\theta\theta}) + \frac{1}{2}(\sigma_{rr} - \sigma_{\theta\theta}) \cos 2\alpha$$

$$= p \left(1 - \frac{a^2}{r^2} \cos 2\alpha \right)$$

$$\tau = -\frac{1}{2}(\sigma_{rr} - \sigma_{\theta\theta}) \sin 2\alpha$$

$$= p \frac{a^2}{r^2} \sin 2\alpha$$

- Slip does not occur if $\phi > 24^\circ$



Excavation shape and boundary stresses

Elliptic opening -Boundary stresses around a mine opening can be established from the elastic solution for particular problem geometry even in presence of discontinuities.

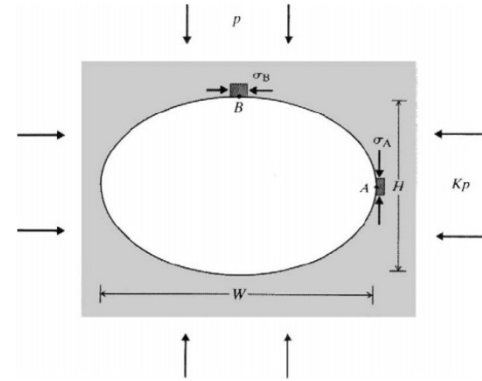
$$\sigma_A = p(1 - K + 2q) = p \left(1 - K + \sqrt{\frac{2W}{\rho_A}} \right)$$

$$\sigma_B = p \left(K - 1 + \frac{2K}{q} \right) = p \left(K - 1 + K \sqrt{\frac{2H}{\rho_B}} \right)$$

Where

- σ boundary stresses
- p radius of curvature
- $q = W/H$

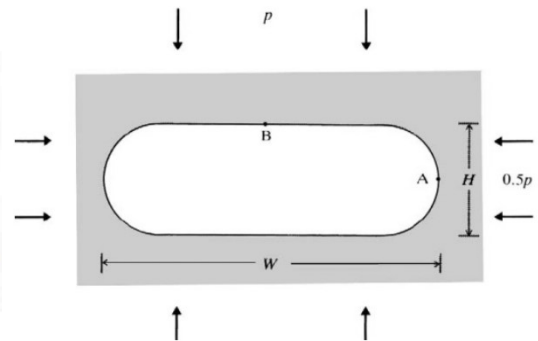
- Larger curvature makes higher stress concentration.



Ovaloidal opening -Applying the boundary stress of an ellipse inscribed in the ovaloid. The width/height ratio for the openings is three, and the radius of curvature for the sidewall is $H/2$. For a ratio of 0.5 of the horizontal and vertical field principle stresses.

$$\sigma_A = p \left(1 - 0.5 + \sqrt{\frac{2 \times 3H}{H/2}} \right) = 3.96p$$

$$\sigma_B = p \left(0.5 - 1 + 0.5 \sqrt{\frac{2H}{(3H)^2/2H}} \right) = -0.17p$$

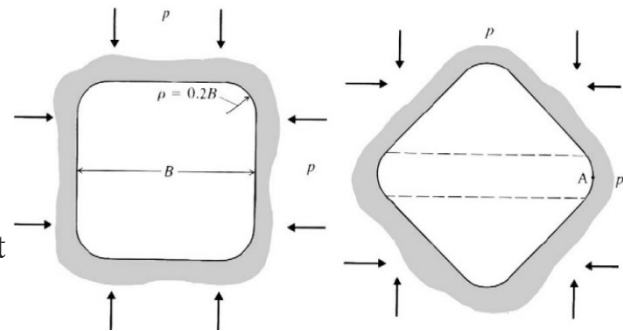


Square opening with rounded corners –Applying the boundary stress of an ellipse whose curvature is the same as those of the rounded corners. The inscribed ovaloid has a width of $2B[2^{1/2} - 0.4(2^{1/2} - 1)]$, from the simple geometry. The boundary stress at the rounded corner is estimated as -

$$\sigma_A = p \left\{ 1 - 1 + \left[\frac{2B(2^{1/2} - 0.4(2^{1/2} - 1))}{0.2B} \right]^{1/2} \right\}$$

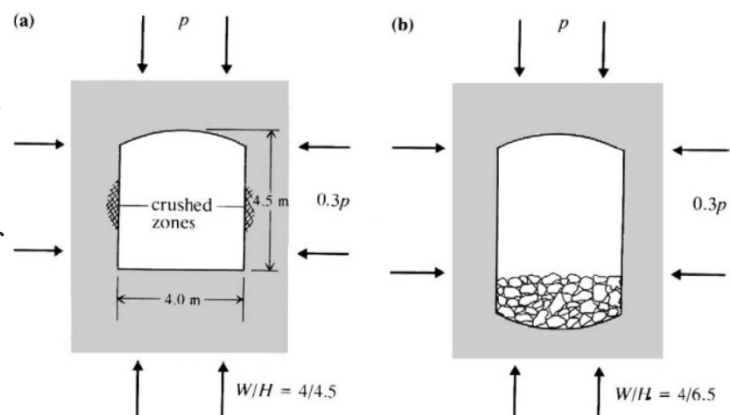
$$= 3.53p$$

The corresponding boundary element solution is $3.14p$.



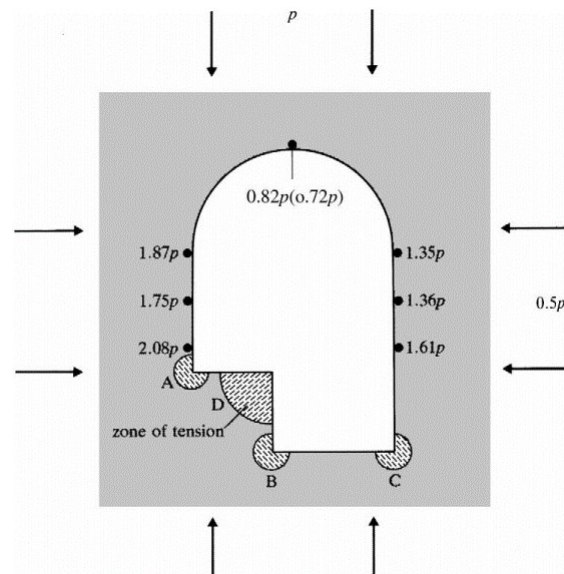
Effect of changing the relative dimensions

- Sidewall stress $2.5p \rightarrow 1.7p$
- The maximum boundary stress can be reduced if the opening dimension is increased in the direction of the major principle stress.



Effect of local geometry of an opening

- Width/height = 2/3
- A, B, C are highly stressed due to their high level of curvature.
- D is at low state of stress.
- Rock mass in compression may behave as a stable continuum while in a de-stressed state, small loads can cause large displacement of rock units.



Delineation of zones of rock failure

- Estimation of the extent of fracture zones provides a basis for prediction of rock mass performance, modification of excavation design, or assessing support and reinforcement requirement.
 - The solution procedure suggested here examines only the initial, linear component of the problem. For mining engineering purposes, the suggested procedure is usually adequate.
- Extent of boundary failure
 - Applicable compressive strength at boundary is σ_{ci} .
 - Tensile strength of rock mass is taken to be zero.

Case of a circular excavation having σ_{ci} of 16 MPa:

$$\sigma_{\theta\theta} = p[1 + K + 2(1 - K)\cos 2\theta]$$

For a rock mass with a crack initiation stress, σ_{ci} , of 16 MPa (perhaps corresponding to a uniaxial strength of the rock material of about 50 MPa), the data of Figure indicate that compressive failure or spalling of the boundary rock occurs over intervals defined by

$$7.5[1.3 + 1.4 \cos 2\theta] \geq 16 \quad \text{i.e. for } \theta \text{ given by}$$

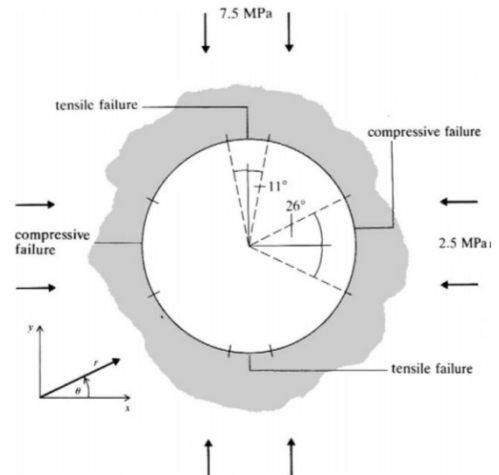
$$26^\circ \leq \theta \leq 26^\circ \text{ or } 154^\circ \leq \theta \leq 206^\circ$$

Similarly, boundary tensile failure occurs over intervals satisfying the condition

$$7.5[1.3 + 1.4 \cos 2\theta] \leq 0$$

$$79^\circ \leq \theta \leq 101^\circ \text{ or } 249^\circ \leq \theta \leq 281^\circ$$

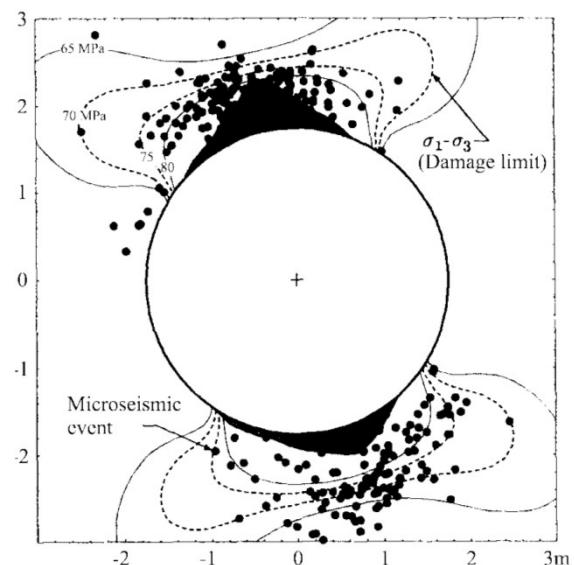
Change in shape, installation of support/reinforcement, or increase the height of the opening can be used.



Extent of failure zones in rock mass

Close to the boundary (within a radius): the constant deviator stress criterion is useful.

Example of an circular opening in Lac du Bonnet granite: the maximum deviator stress contour of 75 MPa predicted well the failure domain.



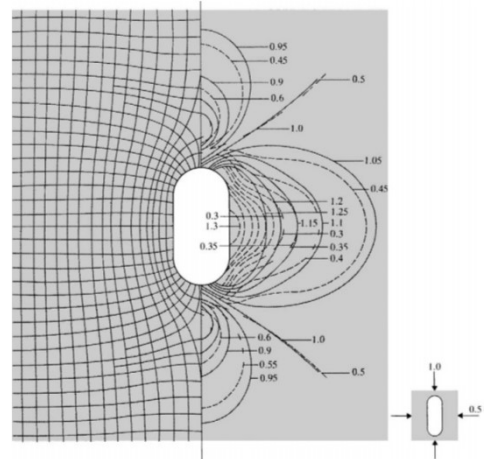
General cases including interior zones of rock mass: Hoek-Brown criterion with σ_{cd}

1) Principal stress contour method

- Calculate various values of (σ_3, σ_1^f)
- Contour plots of s_1 and s_3 are superimposed.
- Find the intersections of s_1 and s_3 isobars satisfying the failure criterion.

2) Direct comparison with the failure criterion

- Calculate the state of stress (principal stress)
- Compare with the failure criterion.
- Display failure locations throughout the rock mass.



Support and reinforcement of massive rock

Explanation of the effect of support

(1) Elastic rock medium

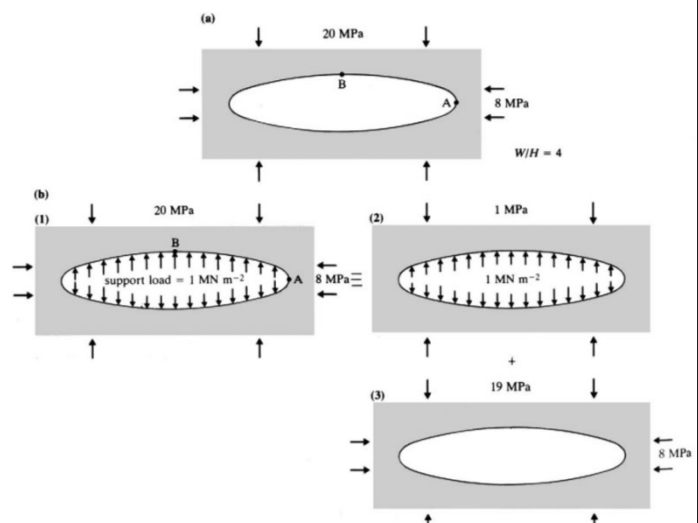
- Stress before support:

$$\sigma_A = 172.0 \text{ MPa}, \sigma_B = -8.0 \text{ MPa}$$

- Stress after support

$$\begin{aligned} \sigma_{A1} &= \sigma_{A2} + \sigma_{A3} \\ &= 1 + 19 \left(1 - \frac{8}{19} + 8 \right) \\ &= 161.0 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_{B1} &= \sigma_{B2} + \sigma_{B3} \\ &= 0 + 19 \left(\frac{8}{19} - 1 + \frac{2 \times 8}{19} \times \frac{1}{4} \right) \\ &= -7.0 \text{ MPa} \end{aligned}$$



Support pressure does not significantly modify the elastic distribution around an underground opening

(2) Elastic rock mass with a failed rock annulus

Rock mass strength is assumed to follow Coulomb's criterion:

$$\sigma_1 = \sigma_3 \frac{(1 + \sin \phi)}{(1 - \sin \phi)} + \frac{2c \cos \phi}{1 - \sin \phi} \quad \text{or} \quad \sigma_1 = b\sigma_3 + C_0$$

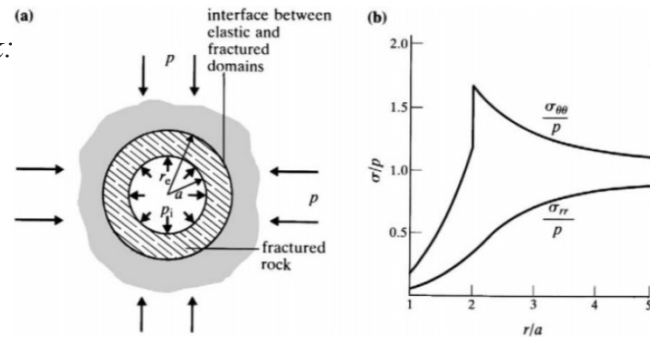
The strength of fractured rock is taken to be purely frictional, with the limiting state of stress within the fractured rock mass defined by

$$\sigma_1 = \sigma_3 \frac{(1 + \sin \phi^f)}{(1 - \sin \phi^f)} \quad \text{or} \quad \sigma_1 = d\sigma_3$$

Equilibrium equations in fractured rock:

Since the problem is axisymmetric, there is only one differential equation of equilibrium-

$$\frac{d\sigma_{rr}}{dr} = \frac{\sigma_{\theta\theta} - \sigma_{rr}}{r} = (d-1) \frac{\sigma_{rr}}{r}$$



Integrating the former expression, and introducing the boundary condition, $\sigma_{rr} = p_i$ when $r = a$, yields the stress distribution relations

$$\sigma_{rr} = p_i \left(\frac{r}{a} \right)^{d-1} \quad \text{and} \quad \sigma_{\theta\theta} = dp_i \left(\frac{r}{a} \right)^{d-1}$$

At the outer limit of the fractured annulus, fractured rock is in equilibrium with intact, elastic rock. If p_1 is the equilibrium radial stress at the annulus outer boundary, r_e ,

$$p_1 = p_i \left(\frac{r_e}{a} \right)^{d-1} \quad \text{or} \quad r_e = a \left(\frac{p_1}{p_i} \right)^{1/(d-1)}$$

Stress in elastic zone: Simple superposition indicates that the stress distribution in the elastic zone is defined by

$$\sigma_{\theta\theta} = p \left(1 + \frac{r_e^2}{r^2} \right) - p_1 \frac{r_e^2}{r^2} \quad \text{and} \quad \sigma_{rr} = p \left(1 - \frac{r_e^2}{r^2} \right) + p_1 \frac{r_e^2}{r^2}$$

At the inner boundary of the elastic zone: when $r = r_e$, the state of stress is defined by

$$\sigma_{\theta\theta} = 2p - p_1 \quad \text{and} \quad \sigma_{rr} = p_1$$

Applying to Coulomb's criterion: This state of stress must represent the limiting state of intact rock (i.e. $\sigma_1 = bp_1 + C_0$), substituting for $\sigma_{\theta\theta}(\sigma_1)$ and $\sigma_{rr}(\sigma_3)$ -

$$2p - p_1 = bp_1 + C_0 \quad \text{or} \quad p_1 = \frac{2p - C_0}{1 + b}$$

Substituting the with annulus outer boundary equation, we get

$$r_e = a \left[\frac{2p - C_0}{(1 + b)p_i} \right]^{1/(d-1)}$$

At the inner boundary of the elastic zone: when $r = r_e$, the state of stress is defined by

and

The equations below, together with support pressure, field stresses and rock properties, completely define the stress distribution and fracture domain in the periphery of the opening.

$$\sigma_{rr} = p_i \left(\frac{r}{a} \right)^{d-1} \quad \sigma_{\theta\theta} = dp_i \left(\frac{r}{a} \right)^{d-1} \quad p_1 = p_i \left(\frac{r_e}{a} \right)^{d-1} \quad r_e = a \left(\frac{p_1}{p_i} \right)^{1/(d-1)}$$

$$r_e = a \left[\frac{2p - C_0}{(1 + b)p_i} \right]^{1/(d-1)}$$

A numerical example provides some insight into the operational function installed support. Choosing particular values of ϕ and ϕ^f of 35° , $p_i = 0.05p$ and $C_0 = 0.5p$, leads to $r_e = 1.99a$.