Contents

Space and Time in Newtonian Relativity

Isaac Newton founded classical mechanics on the view that *space* is distinct from body and that *time* passes uniformly without regard to whether anything happens in the world. For this reason, he spoke of *absolute space* and *absolute time*, so as to distinguish these entities from the various ways by which we measure them (which he called *relative spaces* and *relative times*).

The idea of empty space is a conceptual impossibility. Space is nothing but an abstraction we use to compare different arrangements of the bodies constituting the plenum. There can be no lapse of time without change occurring somewhere. Time is merely a measure of cycles of change within the world.

To paraphrase:

- Absolute, true, and mathematical **time**, from its own nature, passes evenly without relation to anything external, and thus without reference to any change or way of measuring of time (e.g., the hour, day, month, or year).
- Absolute, true, and mathematical **space** remains similar and immovable without relation to anything external. (The specific meaning of this will become clearer below from the way it contrasts with Descartes' concept of space.) Relative spaces are measures of absolute space defined with reference to some system of bodies or another, and thus a relative space may, and likely will, be in motion.
- The **place** of a body is the space which it occupies, and may be absolute or relative according to whether the space is absolute or relative.
- Absolute **motion** is the translation of a body from one absolute place to another; relative motion the translation from one relative place to another.

Newton gives the main part of the Scholium (*a marginal note or explanatory comment made by a scholiast*) to arguing that the distinction between the true quantities and their relative measures is necessary and justified.

It is evident from these characterizations that, according to Newton:

1. space is something distinct from body and exists independently of the existence of bodies,
2. there is a fact of the matter whether a given body moves and what its true quantity of motion is, and
3. the true motion of a body does not consist of, or cannot be defined in terms of, its motion relative to other bodies.

Event: *An event is something that happens independently of the reference frame we might use to describe it. For concreteness, we can imagine the event to be a collision of two particles or the turning-on of a tiny light source. The event happens at a point in space and at an instant in time.*

Description of an event: *We specify an event by four (space-time) measurements in a particular frame of reference, say the position numbers x, y, z and the time t. For example, the collision of two particles may occur at x = 1 m, y = 4 m, z = 11 m, and at time t = 7 sec in one frame of reference (e.g., a laboratory on earth) so that the four numbers (1, 4, 11, 7) specify the event in that reference frame. The same event observed from a different reference frame (e.g., an airplane flying overhead) would also be specified by four numbers, although the numbers may be different than those in the laboratory frame. Thus, if we are to describe events, our first step is to establish a frame of reference.*

Inertial and Noninertial Frame

We define an inertial system as a frame of reference in which the law of inertia—Newton's first law—holds. In such a system, which we may also describe as an unaccelerated system, a body that is acted on by zero net external force will move with a constant velocity.
Newton assumed that a frame of reference fixed with respect to the stars is an inertial system. A rocket ship drifting in outer space, without spinning and with its engines cut off, provides an ideal inertial system.

Frames accelerating with respect to such a system are not inertial. In practice, we can often neglect the small (acceleration) effects due to the rotation and the orbital motion of the earth and to solar motion. Thus, we may regard any set of axes fixed on the earth as forming (approximately) an inertial coordinate system. Likewise, any set of axes moving at uniform velocity with respect to the earth, as in a train, ship, or airplane, will be (nearly) inertial because motion at uniform velocity does not introduce acceleration. However, a system of axes which accelerates with respect to the earth, such as one fixed to a spinning merry-go-round or to an accelerating car, is not an inertial system. A particle acted on by zero net external force will not move in a straight line with constant speed according to an observer in such noninertial systems. The special theory of relativity, which we consider here, deals only with the description of events by observers in inertial reference frames. The objects whose motions we study may be accelerating with respect to such frames but the frames themselves are unaccelerated. The general theory of relativity, presented by Einstein in 1917, concerns itself with all frames of reference, including noninertial ones.

Galilean Transformations

Consider now an inertial frame S and another inertial frame S' which moves at a constant velocity v with respect to S, as shown in Fig. 1. For convenience, we choose the three sets of axes to be parallel and allow their relative motion to be along the common x, x' axis. We can easily generalize to arbitrary orientations and relative velocity of the frames later, but the physical principles involved are not affected by the particular simple choice we make at present. Note also that we can just as well regard S to be moving with velocity -v with respect to S' as we can regard S' to move with velocity v with respect to S.

Let an event occur at point P, whose space and time coordinates are measured in each inertial frame. An observer attached to S specifies by means of meter sticks and clocks, for instance, the location and time of occurrence of this event, ascribing space coordinates x, y, and z and time t to it. An observer attached to S', using his measuring instruments, specifies the same event by space-time coordinates x', y', z', and t'.

The coordinates x, y, z will give the position of P relative to the origin 0 as measured by observer S, and t will be the time of occurrence of P that observer S records with his clocks. The coordinates x', y', and z' likewise refer the position of P to the origin 0' and the time of P, t', to the clocks of inertial observer S'.

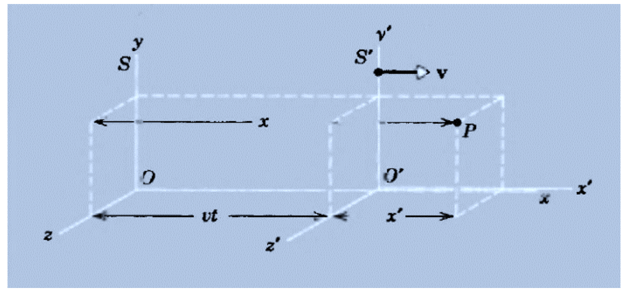


Fig. 1. Two inertial frames with a common x-x' axis and with the y-y' and z z' axes parallel. As seen from frame 5, frame St is moving in the positive x-direction at speed v.

We now ask what the relationship is between the measurements x, y, z, t and x', y', z', t'. The two inertial observers use meter sticks, which have been compared and calibrated against one another, and clocks, which have been synchronized and calibrated against one another. <u>The classical procedure is to assume thereby that length intervals and time intervals are absolute</u>, that is, that they are the same for all inertial observers of the same events. For example, if meter sticks are of the same length when compared at rest with respect to one another, it is implicitly assumed that they are of the same length when compared in relative motion to one another. Similarly, if clocks are calibrated and synchronized when at rest, it is assumed that their readings and rates will agree thereafter, even if they are put in relative motion with respect to one another. These are examples of the common sense" assumptions of classical theory.

We can show these results explicitly, as follows. For simplicity, let us say that the clocks of each observer read zero at the instant that the origins 0 and 0' of the frames S and S', which are in relative motion, coincide. Then the Galilean coordinate transformations, which relate the measurements x, y, z, t to x', y', z', t', are

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

These equations agree with our classical intuition, the basis of which is easily seen from Fig. 1. It is assumed that time can be defined independently of any particular frame of reference. This is an implicit assumption of classical physics, which is expressed in the transformation equations by the absence of a transformation for t. We can make this assumption of the universal nature of time explicit by adding to the Galilean transformations the equation

$$t' = t$$

It follows at once from these Eqs. that the time interval between occurrence of two given events, say P and Q, is the same for each observer, that is

$$t_P' - t_Q' = t_P - t_Q$$

and that the distance, or space interval, between two points, say A and B, measured at a given instant, is the same for each observer, that is

$$x_B' - x_A' = x_B - x_A$$

This result is worth a more careful look. Let A and B be the end points of a rod, for example, which is at rest in the S-frame. Then, the primed observer, for whom the rod is moving with velocity -v, will measure the end-point locations as x_B' and x_A', whereas the unprimed observer locates them at x_B and x_A. Using the Galilean transformations, however, we find that

$$x_B' = x_B - vt_B \qquad \text{and} \qquad x_A' = x_A - vt_A$$

so that

$$x_B' - x_A' = x_B - x_A - v(t_B - t_A)$$

Since the two end points, A and B, are measured at the same instant, $t_B = t_A$ and we obtain $x_B' - x_A' = x_B - x_A$, as found above. Or, we can imagine the rod to be at rest in the primed frame, and moving therefore with velocity v with respect to the unprimed observer. Then the Galilean transformations, which can be written equivalently as

$$x = x' + vt$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

Which give us $x_B - x_A = x_B' - x_A'$.

Notice carefully that two measurements (the end points x_B', x_A' *or* x_B, x_A) are made for each observer and that we assumed they were made at the same time ($t_A = t_B$, *or* $t_A' = t_B'$). The assumption that the measurements are made at the same time—that is, simultaneously—is a crucial part of our definition of the length of the moving rod. Surely we should not measure the locations of the end points at different times to get the length of the moving rod; it would be like measuring the location of the tail of a swimming fish at one instant and of its head at another instant in order to determine its length (see Fig. 1-2).

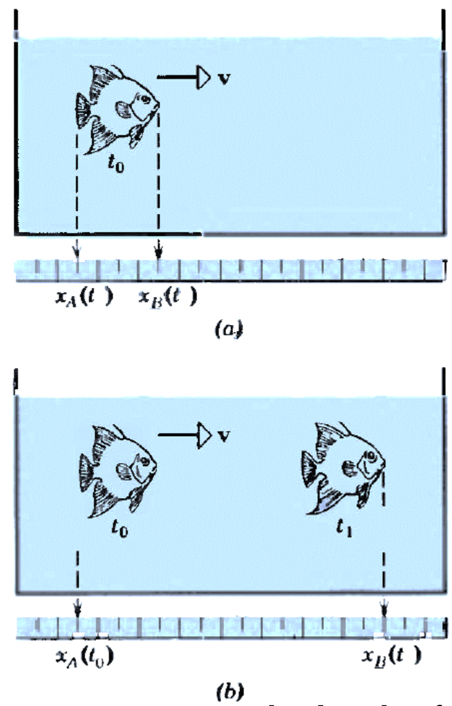


Fig 2. To measure the length of a swimming fish one must mark the positions of its head and tail simultaneously (a), rather than at arbitrary times (b)

The time-interval and space-interval measurements made above are absolutes according to the Galilean transformation; that is, they are the same for all inertial observers, the relative velocity v of the frames being arbitrary and not entering into the results. When we add to this result the assumption of classical physics that the mass of a body is a constant, independent of its motion with respect to an observer, then we can conclude that classical mechanics and the Galilean transformations imply that length, mass, and time—the three basic quantities in mechanics are all independent of the relative motion of the measurer (or observer).

Fig 1-2. To measure the length of a swimming fish one must mark the positions of its head and tail simultaneously (a), rather than at arbitrary times (b)

Newtonian Relativity

Galilean invariance or Galilean relativity states that the laws of motion are the same in all inertial (or non-accelerating) frames. Galileo Galilei first described this principle in 1632 using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. The fact that the Earth orbits around the sun at approximately 30 km/s offers a somewhat more dramatic example, though it is technically not an inertial reference frame.

Specifically, the term Galilean invariance today usually refers to this principle as applied to Newtonian mechanics—that is, Newton's laws hold in all inertial frames. In this context it is sometimes called Newtonian relativity. Among the axioms from Newton's theory are:

- There exists an absolute space in which Newton's laws are true. An inertial frame is a reference frame in relative uniform motion to absolute space.
- All inertial frames share a universal (or absolute) time.

According to Newton, absolute space, in its own nature, without regard to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies: and which is vulgarly taken for immovable space. Absolute motion is the translation of a body from one absolute place into another: and relative motion, the translation from one relative place into another.

Absolute time exists independently of any perceiver (observer) and progresses at a consistent rate throughout the universe. Unlike relative time, Newton believed absolute time was imperceptible and could only be understood mathematically. According to Newton, humans are only capable of perceiving relative time, which is a measurement of perceivable objects in motion (like the Moon or Sun). From these movements, we infer the passage of time.

The position of a particle in motion is a function of time, so that we can express particle velocity and acceleration in terms of time derivatives of position. We need only carry out successive time differentiations of the Galilean trans-formations. The velocity transformation follows at once. Starting from

$$x' = x - vt$$

differentiation with respect to t gives

$$\frac{dx'}{dt} = \frac{dx}{dt} - v$$

But, because t = t', the operation d/dt is identical to the operation d/ dt', so that

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

Similarly

$$\frac{dy'}{dt'} = \frac{dy}{dt}$$

$$\frac{dz'}{dt'} = \frac{dz}{dt}$$

However, $\frac{dx'}{dt'} = u'_x$, the x-component of the velocity measured in S', and $\frac{dx}{dt} = u_x$, the x-component of the velocity measured in S, and so on, so that we have simply the classical velocity addition theorem

$$u'_x = u_x - v$$

$$u'_y = u_y$$

$$u'_z = u_z$$

Clearly, in the more general case in which v, the relative velocity of the frames, has components along all three axes, we would obtain the more general (vector) result

$$u' = u - v$$

The student has already encountered many examples of this. For example, the velocity of an airplane with respect to the air (u') equals the velocity of the plane with respect to the ground (u) minus the velocity of the air with respect to the ground (v).

To obtain the acceleration transformation we merely differentiate the velocity relations. Proceeding as before, we obtain

$$\frac{du'_x}{dt'} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt}$$

v being a constant

$$\frac{du'_y}{dt'} = \frac{du_y}{dt}$$

$$\frac{du'_z}{dt'} = \frac{du_z}{dt}$$

That is,

$$a'_x = a_x$$

$$a'_y = a_y$$

$$a'_z = a_z$$

The measured components of acceleration of a particle are unaffected by the uniform relative velocity of the reference frames.

We have seen that different velocities are assigned to a particle by different observers when the observers are in relative motion. These velocities always differ by the relative velocity of the two observers, which in the case of inertial observers is a constant velocity. It follows then that when the particle velocity changes, the change will be the same for both observers. Thus, they each measure the same acceleration for the particle. The acceleration of a particle is the same in all reference frames which move relative to one another with constant velocity; that is

$$a' = a$$

In classical physics the mass is also unaffected by the motion of the reference frame. Hence, the product ma will be the same for all inertial observers. If $F = ma$ is taken as the definition of force, then obviously each observer obtains the same measure for each force. With $F = ma$ and $F' = ma'$ it follows that $F = F'$.

Newton's laws of motion and the equations of motion of a particle would be exactly the same in all inertial systems. Since, in mechanics, the conservation principles—such as those for energy, linear momentum, and angular momentum—all can be shown to be consequences of Newton's laws, it follows that the laws of mechanics are the same in all inertial frames. Let us make sure that we understand just what this paragraph says, and does not say, before we draw some important conclusions from it:

First, concerning the invariance of Newton's laws (that is, the statement that they are the same for all inertial observers), we should recall that a complete statement of the laws includes the assertions

(1) that particles interact in pairs (third law) and

(2) that the action-reaction forces are directed along the straight line connecting the interacting particles. For many forces that we deal with, it is also true that their magnitude is a function only of the separation of the particles. Thus, these laws apply to such phenomena as gravitation, Van der Waals' forces, and electrostatics. Furthermore, by considering a collection of interacting mass points, we can include the mechanics of rigid bodies, of elastic bodies, and hydrodynamics. Notice, however, that electrodynamics is not included because the interaction between moving electric charges (that is, between charges and magnetic fields) involves forces whose directions are not along the line connecting the charges; notice too, that these forces depend not only on the positions of the charges but also on their velocities.

Second, although different inertial observers will record different velocities for the same particle, and hence different momenta and kinetic energies, they will agree that momentum is conserved in a collision or not conserved, that mechanical energy is conserved or not conserved, and so forth. The tennis ball on the court of a moving ocean liner will have a different velocity to a passenger than it has for an observer on shore, and the billiard balls on the table in a home will have different velocities to the player than they have for an observer on a passing train. But, whatever the values of the particle's or system's momentum or mechanical energy may be, when one observer finds that they do not change in an interaction, the other observer will find the same thing. Although the numbers assigned to such things as velocity, momentum, and kinetic energy may be different for different inertial observers, the laws of mechanics (e.g., Newton's laws and the conservation principles) will be the same in all inertial systems.

Example 1. A passenger walks forward along the aisle of a train at a speed of 2.2 mi/hr as the train moves along a straight track at a constant speed of 57.5 mi/hr with respect to the ground. What is the passenger's speed with respect to the ground?

Ans: Let us choose the train to be the primed frame so that u,' = 2.2 mi/hr. The primed frame moves forward with respect to the ground (unprimed frame) at a speed v = 57.5 mi/hr. Hence, the passenger's speed with respect to ground is

$$u_x = u_x' + v$$

= 2.2 mi/hr +57.5 mi/hr = 59.7 mi/hr.

Example 2. Two electrons are ejected in opposite directions from radioactive atoms in a sample of radioactive material at rest in the laboratory. Each electron has a speed 0.67c as measured by a laboratory observer. What is the speed of one electron as measured from the other, according to the classical velocity addition theorem?

Ans: Here, we may regard one electron as the S frame, the laboratory as the S' frame, and the other electron as the object whose speed in the S-frame is sought (see Fig). In the S'-frame, the other electron's speed is 0.67c, moving in the positive x'-direction say, and the speed of the S-frame (one electron) is 0.67c, moving in the negative x'-direction. Thus, u,' = +0.67c and v = +0.67c, so that the other electron's speed with respect to the S-frame is

$$u_x = u_x' + v$$

= +0.67c + 0.67c = + 1.34c,

according to the classical velocity addition theorem. $

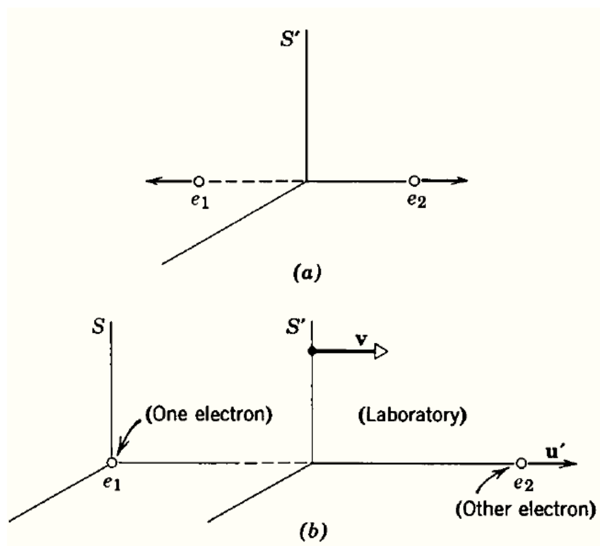
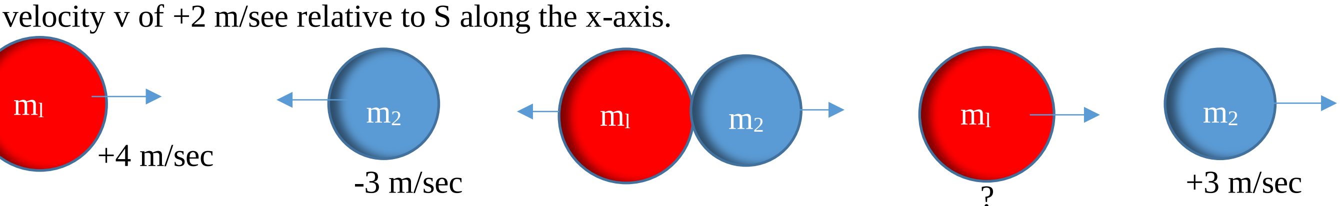


Fig 1-3. (a) In the laboratory frame, the electrons are observed to move in opposite directions at the same speed. (b) In the rest frame, S. of one electron, the laboratory moves at a velocity v. In the laboratory frame, S', the second electron has a velocity denoted by u'

Example 3. A particle of mass m_l = 3 kg, moving at a velocity of u_1 = +4 m/sec along the x-axis of frame S, approaches a second particle of mass m_2 = 1 kg, moving at a velocity u_2 = -3 m/sec along this axis. After a head-on collision, it is found that m_2 has a velocity U_2 = +3 m/sec along the x-axis. (a) Calculate the expected velocity U_1 of m_l, after the collision. (b) Discuss the collision as seen by observer S' who has a velocity v of +2 m/see relative to S along the x-axis.



(a) We use the law of conservation of momentum. Before the collision the momentum of the system of two particles is

$$P = m_1u_1 + m_2u_2 = (3\text{ kg})(+4\text{ m/sec}) + 1\text{ kg }(-3\text{ m/sec}) = +9\text{ kg-m/sec}.$$

After the collision the momentum of the system,

$$P = m_1U_1 + m_2U_2$$

is also +9 kg-m/sec, so that

$$+9\text{ kg-m/sec} = (3\text{ kg})(U_1) + 1\text{ kg }(+3\text{ m/sec})$$

or U_1 = +2 m/see along the x-axis.

(b) The four velocities measured by S' can be calculated from the Galilean velocity transformation equation,
$u' = u - v$, from which we get

$$u_1' = u_1 - v = 4 - 2 = 2\ m/sec$$
$$u_2' = u_2 - v = -3 - 2 = -5\ m/sec$$
$$U_1' = U_1 - v = 2 - 2 = 0\ m/sec$$
$$U_2' = U_2 - v = 3 - 2 = 1\ m/sec$$

The system momentum in S' before the collision is $P' = m_1u'_1 + m_2u'_2$

$$= (3\text{ kg})(2\text{ m/sec}) + (1\text{ kg})(-5\text{ m/sec}) = +1\text{ kg-m/sec},$$

and after the collision

$$P' = m_1U'_1 + m_2U'_2$$

$$= (3\text{ kg})(0) + (1\text{ kg})(1\text{ m/sec}) = +1\text{ kg-m/sec} \quad .$$

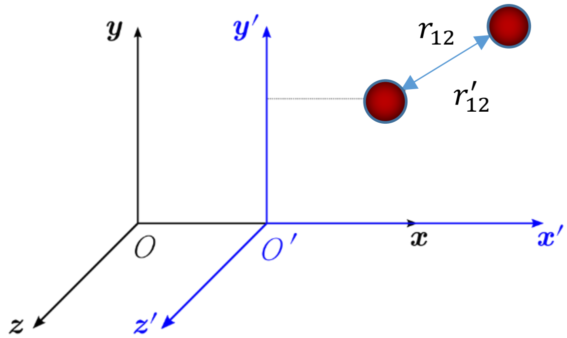
Hence, although the velocities and momenta have different numerical values in the two frames, S and S', when momentum is conserved in S it is also conserved in S'

An important consequence of the above discussion is that no mechanical experiments carried out entirely in one inertial frame can tell the observer what the motion of that frame is with respect to any other inertial frame. The billiard player in a closed box-car of a train moving uniformly along a straight track cannot tell from the behavior of the balls what the motion of the train is with respect to ground. The tennis player in an enclosed court on an ocean liner moving with uniform velocity (in a calm sea) cannot tell from his game what the motion of the boat is with respect to the water. No matter what the relative motion may be (perhaps none), so long as it is constant, the results will be identical. Of course, we can tell what the relative velocity of two frames may be by comparing measurements between frames—we can look out the window of a train or compare the data different observers take on the very same event—but then we have not deduced the relative velocity from observations confined to a single frame.

Furthermore, there is no way at all of determining the absolute velocity of an inertial reference frame from our mechanical experiments. No inertial frame is preferred over any other, for the laws of mechanics are the same in all. Hence, there is no physically definable absolute rest frame. We say that all inertial frames are equivalent as far as mechanics is concerned. The person riding the train cannot tell absolutely whether he alone is moving, or the earth alone is moving past him, or if some combination of motions is involved. Indeed, would you say that you on earth are at rest, that you are moving 30 km/sec (the speed of the earth in its orbit about the sun) or that your speed is much greater still (for instance, the sun's speed in its orbit about the galactic center)? Actually, no mechanical experiment can be performed which will detect an absolute velocity through empty space. This result, that we can only speak of the relative velocity of one frame with respect to another, and not of an absolute velocity of a frame, is sometimes called Newtonian relativity.

Example 4. Consider the forces that two particles exert on each other to lie along their connecting straight line, the magnitude of these equal and opposite forces being a function only of the separation distance of the particles. Under these conditions, forces can always be represented by the negative space derivatives of the potential energy. Show that the equation of motion of such a particle remains unchanged under a Galilean transformation.

Ans:



Let the distance between the two particles be r_{12} in the S frame, and r_{12}' in the S' frame. Then the potential energy U of the system in S will be a function of r_{12}, which we write as U(r_{12}). Hence, the components of force are given by

$$F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}, \quad F_z = -\frac{\partial U}{\partial z}$$

The equations of motion in frame S for particle 1, say, of mass m_1 will therefore be

$$\left.\begin{aligned} m_1 \frac{d^2 x_1}{dt^2} &= -\frac{\partial U}{\partial x_1} \\ m_1 \frac{d^2 y_1}{dt^2} &= -\frac{\partial U}{\partial y_1} \\ m_1 \frac{d^2 z_1}{dt^2} &= -\frac{\partial U}{\partial z_1} \end{aligned}\right\} \qquad (1)$$

Now the mass of a body is assumed to be independent of the inertial reference frame in which it is measured in classical physics. Also, we have seen that under a Galilean transformation the S' observer obtains the same acceleration for a body as the S observer does. Hence (using x', y', z', and t' for the primed observer's variables to describe the motion of the same particle that the unprimed observer described with x, y, z, and t), we have already found that $m_1' = m_1$, and that

$$\frac{d^2x_1}{dt^2} = \frac{d^2x_1'}{dt'^2}; \qquad \frac{d^2y_1}{dt^2} = \frac{d^2y_1'}{dt'^2}; \qquad \frac{d^2z_1}{dt^2} = \frac{d^2z_1'}{dt'^2}$$

Furthermore, we have seen that both observers measure the same separation of the two particles. That is,

$$x_2' - x_1' = x_2 - x_1$$

$$y_2' - y_1' = y_2 - y_1$$

$$z_2' - z_1' = z_2 - z_1$$

The potential energy of the system is represented by $U(r_{12})$, which is some function of the separation of the particles, such as $\frac{k}{r_{12}}$ for example. Because $r_{12} = r_{12}'$, hence, $U(r_{12}) = U(r_{12}')$, where $U(r_{12}')$ expresses the potential energy in the primed system's variables. Remember that we are trying to prove that the equations of motion of particle-1 in one inertial frame S will have the identical form as the equations of motion of the same particle in another inertial system S', if the relation between the different observer's variables is given by the Galilean transformation. That is, we are trying to show that each inertial observer uses the same laws of mechanics. So far, we have found that the left side of Eqs.1, when transformed from S to S', has the identical form and that $U(r_{12})$ and $U(r_{12}')$ are identical.

Let us do the x differentiation only (the y and z differentiations proceed identically). We have

$$-\frac{\partial U}{\partial x_1} = -\frac{dU}{dr_{12}}\frac{\partial r_{12}}{\partial x_1} = -\frac{dU}{dr_{12}}\frac{x_2 - x_1}{r_{12}}$$

and

$$-\frac{\partial U}{\partial {x_1}'} = -\frac{dU}{d{r_{12}}'}\frac{\partial {r_{12}}'}{\partial {x_1}'} = -\frac{dU}{d{r_{12}}'}\frac{{x_2}' - {x_1}'}{{r_{12}}'}$$

But $r_{12} = r_{12}'$; $x_2' - x_1' = x_2 - x_1$; and $U(r_{12}) = U(r_{12}')$ so that

$$-\frac{\partial U}{\partial x_1} = -\frac{\partial U}{\partial {x_1}'}$$

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz$$

$$\Rightarrow \frac{dU}{dr_{12}} = \frac{\partial U}{\partial x}\frac{dx}{dr_{12}} \Longrightarrow \frac{\partial U}{\partial x_1}\frac{\partial x_1}{\partial r_{12}}$$

$$\Rightarrow \frac{\partial U}{\partial x_1} = \frac{dU}{dr_{12}}\frac{\partial r_{12}}{\partial x_1}$$

Now, $r_{12}\partial r_{12} = (x_2 - x_1)\partial x_1$

$$\Rightarrow \frac{\partial r_{12}}{\partial x_1} = \frac{x_2 - x_1}{r_{12}}$$

Hence, by applying the Galilean transformation equations to the equations of motion of particle-1 in S, we obtain the identical equations of motion for this same particle-1 in S', namely,

$$m_1\frac{d^2{x_1}'}{dt'^2} = -\frac{\partial U}{\partial {x_1}'}$$

$$m_1\frac{d^2{y_1}'}{dt'^2} = -\frac{\partial U}{\partial {y_1}'}$$

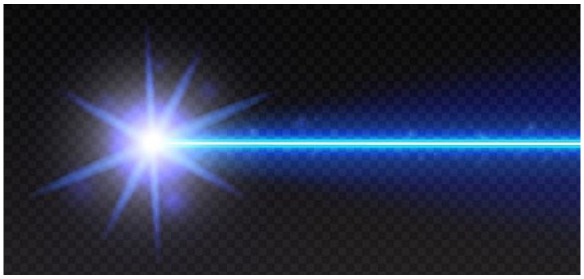
$$m_1 \frac{d^2 z_1{}'}{dt'^2} = -\frac{\partial U}{\partial z_1{}'} \qquad (2)$$

in which the variables x_1, y_1, z_1 and t of S in Eqs.1 simply become the corresponding variables $x_1', y_1{}', z_1{}'$ and t' of S' in Eqs.2. Obviously, we would obtain similar results for particle 2, and indeed the procedure is easily generalized to a large collection of mass particles. This example illustrates explicitly the statement that Newton's laws of mechanics and the equations of motion are the same in all inertial frames when the frames are related by the Galilean transformation equations. Under a Galilean transformation $F = ma$ becomes $F' = ma'$.

Transformation laws, in general, will change many quantities but will leave some others unchanged. These unchanged quantities are called **invariants** of the transformation. In the Galilean transformation laws for the relation between observations made in different inertial frames of reference, for example, acceleration is an invariant and —more important— so are Newton's laws of motion. A statement of what the invariant quantities are is called a relativity principle; it says that for such quantities the reference frames are equivalent to one another, no one having an absolute or privileged status relative to the others. Newton expressed his relativity principle as follows: "The motions of bodies included in a given space are the same amongst themselves, whether that space is at rest or moves uniformly forward in a straight line."

Electromagnetism and Newtonian Relativity

Let us now consider the situation from the electrodynamic point of view. That is, we inquire now whether the laws of physics other than those of mechanics (such as the laws of electromagnetism) are invariant under a Galilean transformation. If so, then the (Newtonian) relativity principle would hold not only for mechanics but for all of physics. That is, no inertial frame would be preferred over any other and no type of experiment in physics, not merely mechanical ones, carried out in a single frame would enable us to determine the velocity of our frame relative to any other frame. There would then be no preferred, or absolute reference frame.



To see at once that the electromagnetic situation is different from the mechanical one, as far as the Galilean transformations are concerned, consider a pulse of light (i.e., an electromagnetic pulse) traveling to the right with respect to the medium through which it is propagated at a speed c. The "medium" of light propagation was given the name "ether," historically, for when the mechanical view of physics dominated physicists' thinking (late 19th century and early 20th century) it was not really accepted that an electromagnetic disturbance could be propagated in empty space. For simplicity, we may regard the "ether" frame, S, as an inertial one in which an observer measures the speed of light to be exactly $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.997925 \times 10^8 \, m/s.$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{(wave eqn.)} \qquad\qquad \frac{\partial^2\psi}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2\psi}{\partial t^2} \quad \text{(For e.m. wave)}$$

In a frame S' moving at a constant speed v with respect to this ether frame, an observer would measure a different speed for the light pulse, ranging from c + v to c — v depending on the direction of relative motion, according to the Galilean velocity transformation. Hence, the speed of light is certainly not invariant under a Galilean transformation. If these transformations really do apply to optical or electromagnetic phenomena, then there is one inertial system, and only one, in which the measured speed of light is exactly c; that is, there is a unique inertial system in which the so-called ether is at rest. We would then have a physical way of identifying an absolute (or rest) frame and of determining by optical experiments carried out in some other frame what the relative velocity of that frame is with respect to the absolute one.

A more formal way of saying this is as follows. Maxwell's equations of electromagnetism, from which we deduce the electromagnetic wave equation for example, contain the constant $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, which is identified as the velocity of propagation of a plane wave in vacuum. But such a velocity cannot be the same for observers in different inertial frames, according to the Galilean transformations, so that electromagnetic effects will probably not be the same for different inertial observers. In fact, Maxwell's equations are not preserved in form by the Galilean transformations, although Newton's laws are. In going from frame S to frame S', the form of the wave equation, for example, changes if the substitutions (of Eqs. GT) are made. But if we accept both the Galilean transformations and Maxwell's equations as basically correct, then it automatically follows that there exists a unique privileged frame of reference (the "ether" frame) in which Maxwell's equations are valid and in which light is propagated at a speed $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$. The situation then seems to be as follows: The fact that the Galilean relativity principle does apply to the Newtonian laws of mechanics but not to Maxwell's laws of electromagnetism requires us to choose the correct consequences from amongst the following possibilities.

1. A relativity principle exists for mechanics, but not for electrodynamics; in electrodynamics there is a preferred inertial frame; that is, the ether frame. Should this alternative be correct the Galilean transformations would apply and we would be able to locate the ether frame experimentally.

2. A relativity principle exists both for mechanics and for electrodynamics, but the laws of electrodynamics as given by Maxwell are not correct. If this alternative were correct, we ought to be able to perform experiments that show deviations from Maxwell's electrodynamics and reformulate the electromagnetic laws. The Galilean transformations would apply here also.

3. A relativity principle exists both for mechanics and for electrodynamics, but the laws of mechanics as given by Newton are not correct. If this alternative is the correct one, we should be able to perform experiments which show deviations from Newtonian mechanics and reformulate the mechanical laws. In that event, the correct transformation laws would not be the Galilean ones (for they are inconsistent with the invariance of Maxwell's equations) but some other ones which are consistent with classical electromagnetism and the new mechanics. We have already indicated that Newtonian mechanics breaks down at high speeds so that the student will not be surprised to learn that alternative 3, leading to Einsteinian relativity, is the correct one. In the following sections, we shall look at the experimental bases

for rejecting alternatives 1 and 2, as a fruitful run-up to finding the new relativity principle and transformation laws of alternative 3.

Luminiferous Aether

It seemed inconceivable to 19th century physicists that light and other electromagnetic waves, in contrast to all other kinds of waves, could be propagated without a medium. It seemed to be a logical step to postulate such a medium, called the ether, even though it was necessary to assume unusual properties for it, such as zero density, perfect transparency, invisible, odorless to account for its undetectability. The ether was assumed to fill all space and to be the medium with respect to which the speed c applies. It followed then that an observer moving through the ether with velocity v would measure a velocity e' for a light beam, where $c' = c + v$. It was this result that the Michelson-Morley experiment was designed to test.

Michelson Morley Experiment

The Michelson interferometer (Fig. 1) is fixed on the earth. If we imagine the "ether" to be fixed with respect to the sun, then the earth (and interferometer) moves through the ether at a speed of 30 km/sec, in different directions in different seasons (Fig. 2). For the moment, neglect the earth's spinning motion. The beam of light (plane waves, or parallel rays) from the laboratory source S (fixed with respect to the instrument) is split by the partially silvered mirror M into two coherent beams, beam 1 being transmitted through M and beam 2 being reflected off of M. Beam 1 is reflected back to M by mirror M_1 and beam 2 by mirror M_2. Then the returning beam 1 is partially reflected and the returning beam 2 is partially transmitted by M back to a telescope at T where they interfere. The interference is constructive or destructive depending on the phase difference of the beams. The partially silvered mirror surface M is inclined at 45° to the beam directions. If M_1 and M_2 are very nearly (but not quite) at right angles, we shall observe a

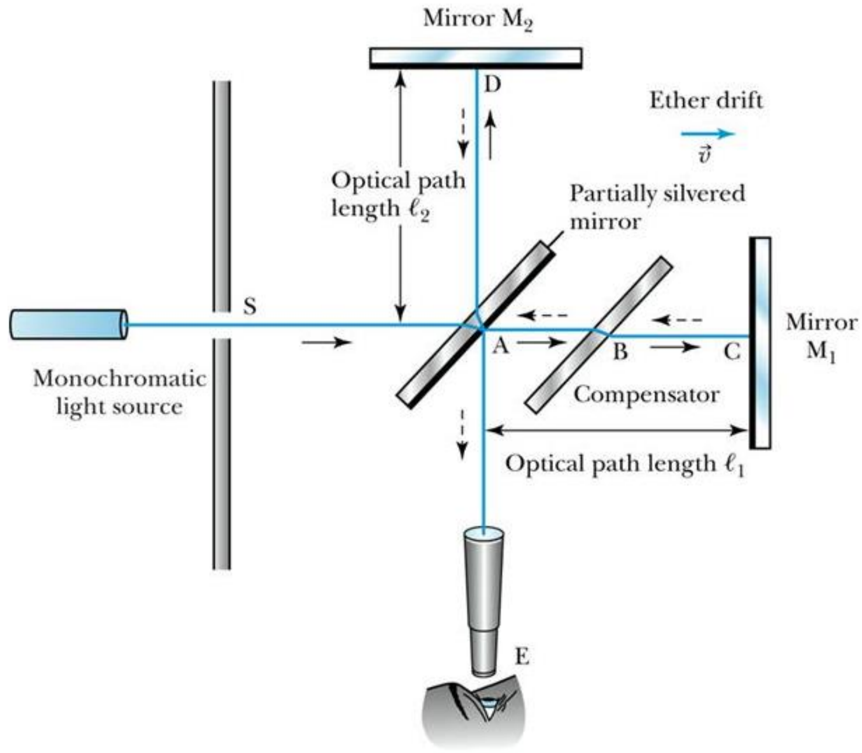


Fig. 1

fringe system in the telescope (Fig. 3) consisting of nearly parallel lines, much as we get from a thin wedge of air between two glass plates.

Let us compute the phase difference between the beams 1 and 2. This difference can arise from two causes, the different path lengths traveled, l_1 and l_2, and the different speeds of travel with respect to the instrument because of the "'ether wind" v. The second cause, for the moment, is the crucial one. The different speeds are much like the different cross-stream and up-and-down-stream speeds with respect to shore of a swimmer in a moving stream. The time for beam 1 to travel from M to M_1 and back is

$$t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v}$$

$$= l_1 \frac{2c}{(c-v)(c+v)}$$

$$= l_1 \frac{2c}{(c^2 - v^2)}$$

$$= \frac{2l_1}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right)$$

for the light, whose speed is c in the ether, has an "upstream" speed of $c - v$ with respect to the apparatus and a "downstream" speed of $c + v$. The path of beam 2, traveling from M to M_2 and back, is a cross-stream path through the ether, as shown in Fig. 4, enabling the beam to return to the (advancing) mirror M. The transit time is given by

$$l_2{}^2 + \left(\frac{vt_2}{2}\right)^2 = \left(\frac{ct_2}{2}\right)^2$$

$$4l_2{}^2 + (vt_2)^2 = (ct_2)^2$$

$$4l_2{}^2 = (c^2 - v^2)(t_2)^2$$

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}}$$

$$t_2 = \frac{2l_2}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

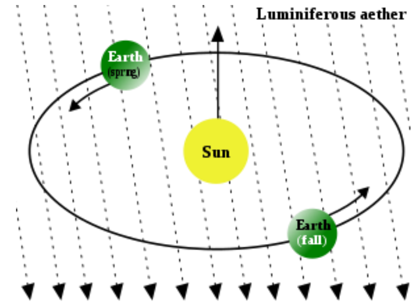


Fig. 2: The earth E moves at an orbital speed of 30 km/sec along its nearly circular orbit about the sun S, reversing the direction of its velocity every six months.

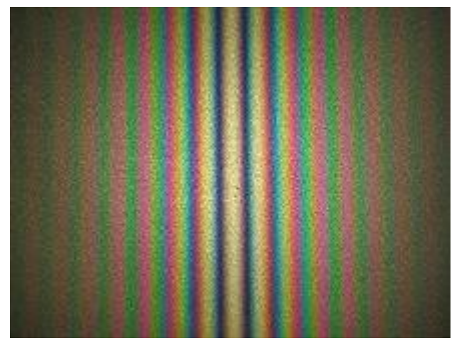


Fig. 3: A typical fringe system seen through the telescope T when M, and M2 are not quite at right angles

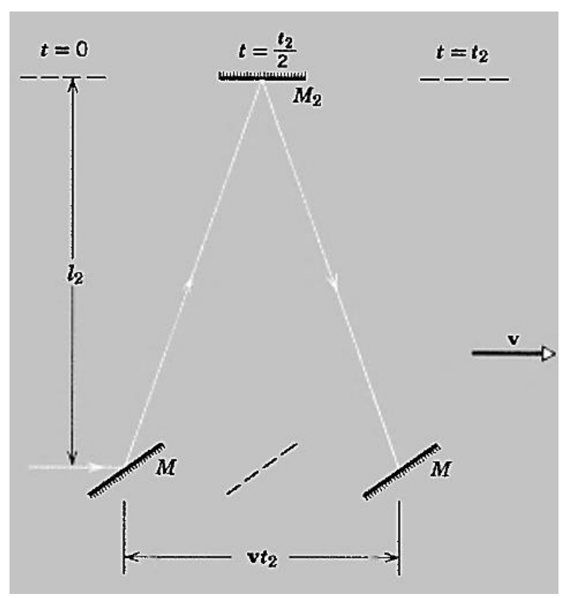


Fig. 4

The calculation of t_2 is made in the ether frame, that of t_1 in the frame of the apparatus. Because time is an absolute in classical physics, this is perfectly acceptable classically. Note that both effects are second-order ones $\left(\frac{v^2}{c^2} \approx 10^8\right)$ and are in the same direction (they increase the transit time over the case v = 0). The difference in transit times is

$$\Delta t = t_2 - t_1 = \left\{\frac{2l_2}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right\} - \left\{\frac{2l_1}{c}\left(\frac{1}{1-\frac{v^2}{c^2}}\right)\right\}$$

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left[\frac{l_2}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{l_1}{1-\frac{v^2}{c^2}}\right]$$

Suppose that the instrument is rotated through 90°, thereby making l_1 the cross-stream length and l_2 the downstream length. If the corresponding times are now designated by primes, the same analysis as above gives the transit-time difference as

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left[\frac{l_2}{1-\frac{v^2}{c^2}} - \frac{l_1}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

Hence, the rotation changes the differences by

$$\Delta t' - \Delta t = \frac{2}{c}\left[\frac{l_2}{1-\frac{v^2}{c^2}} - \frac{l_1}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{l_2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{l_1}{1-\frac{v^2}{c^2}}\right]$$

$$= \frac{2}{c}\left[\frac{l_2+l_1}{1-\frac{v^2}{c^2}} - \frac{l_2+l_1}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{2(l_2 + l_1)}{c}\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

Using the binomial expansion and dropping terms higher than the second-order, we find

$$\Delta t' - \Delta t \cong \frac{2(l_2 + l_1)}{c}\left[1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right]$$

$$= \frac{(l_2 + l_1)}{c}\frac{v^2}{c^2}$$

Therefore, the rotation should cause a shift in the fringe pattern, since it changes the phase relationship between beams 1 and 2. If the optical path difference between the beams changes by one wave- length, for example, there will be a shift of one fringe across the cross- hairs of the viewing telescope. Let ΔN represent the number of fringes moving past the crosshairs as the pattern shifts. Then, if light of wave-length λ is used, so that the period of one vibration is $T = \frac{1}{n} = \frac{\lambda}{c}$

$$\Delta N = \frac{\Delta t' - \Delta t}{T} = = \frac{(l_2 + l_1)}{\lambda}\frac{v^2}{c^2}$$

Michelson and Morley were able to obtain an optical path length, $l_2 + l_1$, of about 22 m. In their experiment the arms were of (nearly) equal length, that is, $l_2 = l_1 = l$, so that $\Delta N = \frac{2l}{\lambda}\frac{v^2}{c^2}$. If we choose $\lambda = 5.5 \times 10^{-7}\, m$ and v/c = 10^{-4}, we obtain, from above Eq.

$$\Delta N = \frac{2l}{\lambda}\frac{v^2}{c^2} = \frac{22}{5.5 \times 10^{-7}} 10^{-4} = 0.4$$

or a shift of four-tenths a fringe! Michelson and Morley mounted the interferometer on a massive stone slab for stability and floated the apparatus in mercury so that it could be rotated smoothly about a central pin. In order to make the light path as long as possible, mirrors were arranged on the slab to reflect the beams back and forth through eight round trips. The fringes were observed under a continuous rotation of the apparatus and a shift as small as 1/100 of a fringe definitely could have been detected. Observations were made day and night (as the earth spins about its axis) and during all seasons of the year (as the earth rotates about the sun), but the expected fringe shift was not observed. Indeed, the experimental conclusion was that there was no fringe shift at all.

This null result (ΔN = 0) was such a blow to the ether hypothesis that the experiment was repeated by many workers over a 50-year period. The null result was amply confirmed and provided a great stimulus to theoretical and experimental investigation. In 1958 J. P. Cedarholm, C. H. Townes et al carried out an “ether-wind” experiment using microwaves in which they showed that if there is an ether and the earth is moving through it, the earth’s speed with respect to the ether would have to be less than 1/1000 of the earth’s orbital speed. This is an improvement of 50 in precision over the best experiment of the Michelson-Morley type. The null result is well established.

The student should note that the Michelson-Morley experiment depends essentially on the 90° rotation of the interferometer, that is, on interchanging the roles of l_2 and l_1 as the apparatus moves with a speed v through an "ether." In predicting an expected fringe shift, we took v to be the earth's velocity with respect to an ether fixed with the sun. However, the solar system itself might be in motion with respect to the hypothetical ether. Actually, the experimental results themselves determine the earth's speed with respect to an ether, if indeed there is one, and these results give v = 0. Now, if at some time the velocity were zero in such an ether, no fringe shift would be expected, of course. But the velocity cannot always be zero, since the velocity of the apparatus is changing from day to night (as the earth spins) and from season to season (as the earth rotates about the sun). Therefore, the experiment does not depend solely on an "absolute" velocity of the earth through an ether, but also depends on the changing velocity of the earth with respect to the "ether." Such a changing motion through the "ether" would be easily detected and measured by the precision experiments, if there were an ether frame. The null result seems to rule out an ether (absolute) frame.

One way to interpret the null result of the Michelson-Morley experiment is to conclude simply that the measured speed of light is the same, that is, c, for all directions in every inertial system. For this fact would lead to $\Delta N = 0$ in the (equal arm) experiment, the "downstream" and "cross-stream" speeds being c, rather than |c + v|, in any frame. However, such a conclusion, being incompatible with the Galilean (velocity) transformations, seemed to be too drastic philosophically at the time. If the measured speed of light did not depend on the motion of the observer, all inertial systems would be equivalent for a propagation of light and there could be no experimental evidence to indicate the existence of a unique inertial system, that is, the ether. Therefore, to "save the ether" and still explain the Michelson-Morley result, scientists suggested alternative hypotheses.

Reference:

- Introduction to special relativity – Robert Resnick