Circuit Analysis

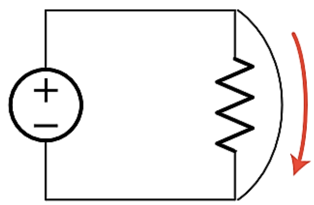
Terminologies

Circuit: Circuit is a collection of real components, power sources, and signal sources, all connected so current can flow in a complete circle.

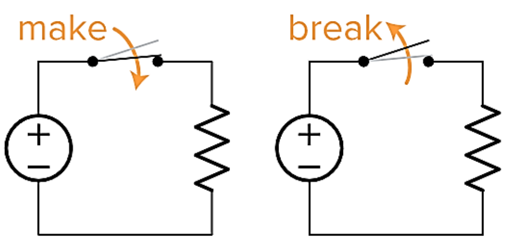
Closed circuit: A circuit is *closed* if the circle is complete, if all currents have a path back to where they came from.

Open circuit: A circuit is *open* if the circle is not complete, if there is a gap or opening in the path.

Short circuit: A *short* happens when a path of low resistance is connected (usually by mistake) to a component. The resistor shown below is the intended path for current, and the curved wire going around it is the short. Current is diverted away from its intended path, sometimes with damaging results. The wire *shorts out* the resistor by providing a low-resistance path for current (probably not what the designer intended).



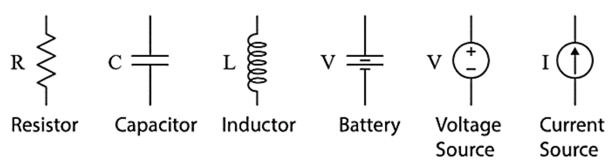
Make or Break: You *make* a circuit by closing the current path, such as when you close a switch. *Breaking* a circuit is the opposite. Opening a switch *breaks* the circuit.



Schematic: A *schematic* is a drawing of a circuit. A schematic represents circuit elements with symbols and connections as lines.

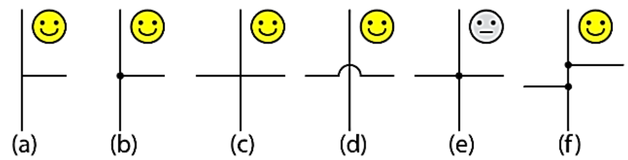
Elements: The term *elements* means "components and sources."

Symbols: Elements are represented in schematics by *symbols*. Symbols for common 2-terminal elements are sh own here,



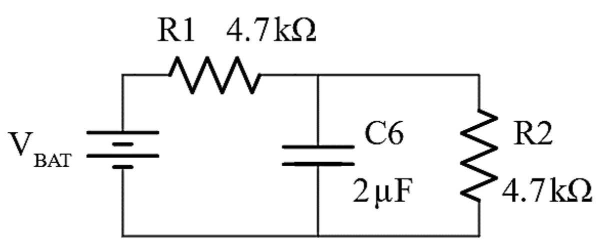
Lines: Connections between elements are drawn as lines, which we often think of as "wires". On a schematic, these lines represent perfect conductors with zero resistance. Every component or source terminal touched by a line is at the same voltage.

Dots: Connections between lines can be indicated by *dots*. Dots are an unambiguous indication that lines are connected. If the connection is obvious, you don't have to use a dot.

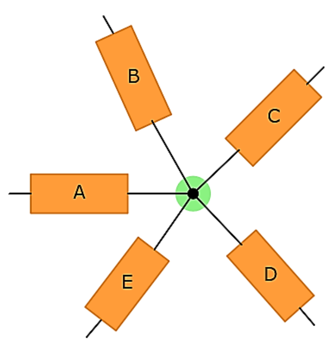


(a) and (b) are both good
(c) no dot indicates no connection
(d) also indicates no connection; the horizontal wire "hops" over the vertical wire. It is very clear but takes extra effort and space to draw.
(e) for crossing connected lines, it is acceptable, but risks looking too much like (c), so (f) is the better practice.

Reference designator: When you place a component in a schematic you often give it a unique name, known as a *reference designator*. Examples of reference designators are R_1, C_6, and V_{BAT}. The 1 in R_1 is part of the name, and does not indicate the resistance value. Reference designators are by definition unique for each schematic. They let you identify components by name even if some of them have the same value. It is okay to use reference designators in equations. R_1 can be assigned a resistance value, R_1=4.7 kΩ and it can be used as a variable in expressions, as in $R_2 \cdot C_6 = 4.7\ k\Omega \cdot 2\ \mu F$.



Node: A junction where two or more elements connect is called a *node*. The schematic below shows a single node (the black dot) formed by the junction of five elements (abstractly represented by orange rectangles).



Since lines on a schematic represent perfect zero-resistance conductors, there is no rule that says lines from multiple elements are required to meet in a single point junction. We can draw the same node as a *distributed* node like the one in the schematic below. These two representations of the node mean exactly the same thing.

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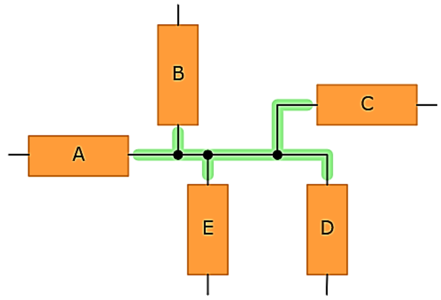
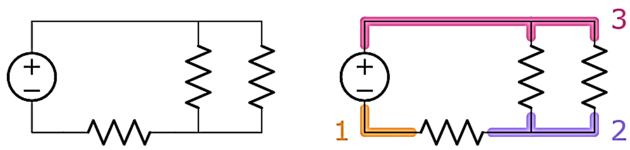
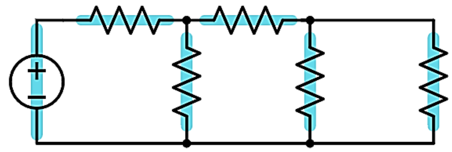


A distributed node might be all spread out, with lots of line segments, elbows, and dots. Don't be distracted, it is all just one single node. Connecting schematic elements with perfect conductors means the voltage everywhere on a distributed node is the same.

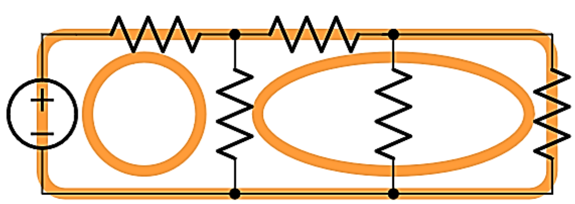
Here is a realistic-looking schematic with the distributed nodes labeled:

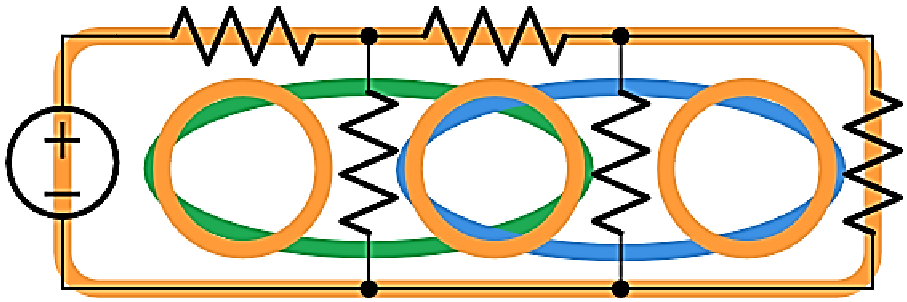


Branch: *Branches* are the connections between nodes. A branch is an element (resistor, capacitor, source, etc.). The number of branches in a circuit is equal to the number of elements.

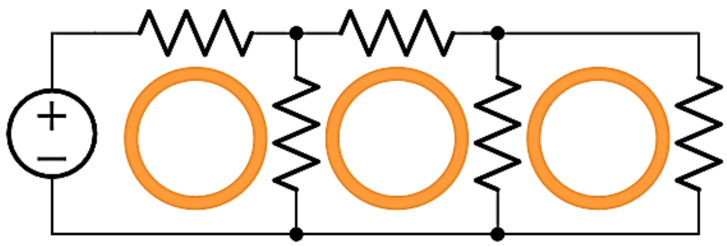


Loop: A *loop* is any closed path going through circuit elements. To draw a loop, select any node as a starting point and draw a path through elements and nodes until the path comes back to the node where you started. <u>There is only one rule: a loop can visit (pass through) a node only *one time*</u>. It is ok if loops overlap or contain other loops. Some of the loops in our circuit are shown here. (You can find others, too. If I counted right, there are six.)





Mesh: A *mesh* is a loop that has no other loops inside it. You can think of this as one mesh for each "open window" of a circuit.



Reference Node: During circuit analysis we usually pick one of the nodes in the circuit to be the *reference node*. Voltages at other nodes are measured relative to the reference node. Any node can be the reference, but two common choices that simplify circuit analysis are,

- the negative terminal of the voltage or current source powering the circuit, or
- the node connected to the greatest number of branches.

Ground: The reference node is often referred to as *ground*. The concept of *ground* has three important meanings. Ground is

- the reference point from which voltages are measured.
- the return path for electric current back to its source.
- a direct physical connection to the Earth, which is important for safety.

Impedance: Impedance (symbol Z) is a measure of the overall opposition of a circuit to current, in other words: how much the circuit **impedes** the flow of charge. It is like resistance, but it also takes into account the effects of capacitance and inductance. Impedance is measured in ohms (Ω).

Impedance is more complex than resistance because the effects of capacitance and inductance vary with the frequency of the current passing through the circuit and this means **impedance varies with frequency**. The effect of resistance is constant regardless of frequency.

$$Z = \sqrt{R^2 + X^2}$$

$$X = X_L - X_C$$

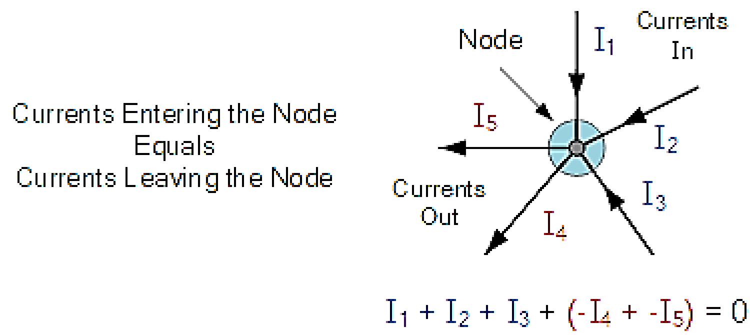
Previous knowledge

Kirchhoff's Law and its Applications

Kirchhoff's circuit laws are two equalities that deal with the current and potential difference (commonly known as voltage) in the lumped element model of electrical circuits. They were first described in 1845 by German physicist Gustav Kirchhoff.

Kirchhoff's First Law – The Current Law, (KCL)

Kirchhoff's Current Law or KCL, states that the "total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node''. In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero, $I_{(exiting)} + I_{(entering)} = 0$. This idea by Kirchhoff is commonly known as the Conservation of Charge.



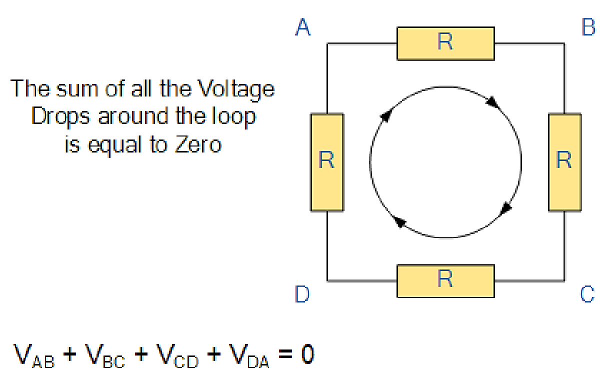
Here, the three currents entering the node, I_1, I_2, I_3 are all positive in value and the two currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

$I_1 + I_2 + I_3 – I_4 – I_5 = 0$

The term Node in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchhoff's current law when analyzing parallel circuits.

Kirchhoff's Second Law – The Voltage Law, (KVL)

Kirchhoffs Voltage Law or KVL, states that "in any closed loop network, the directional sum of the voltage drops in various components in the loop is equal to the directional sum of the e.m.f.'s of the voltage source in the same network". In other words the algebraic sum of all voltages within the loop must be equal to zero. This idea by Kirchhoff is known as the Conservation of Energy.



Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchhoff's voltage law when analyzing series circuits.

I

Superposition Theorem

"In any linear network containing impedances and energy sources, the current flowing in any element is the vector sum of the currents that are separately caused to flow in that element by each energy source".

$$I = I_1 + I_2$$

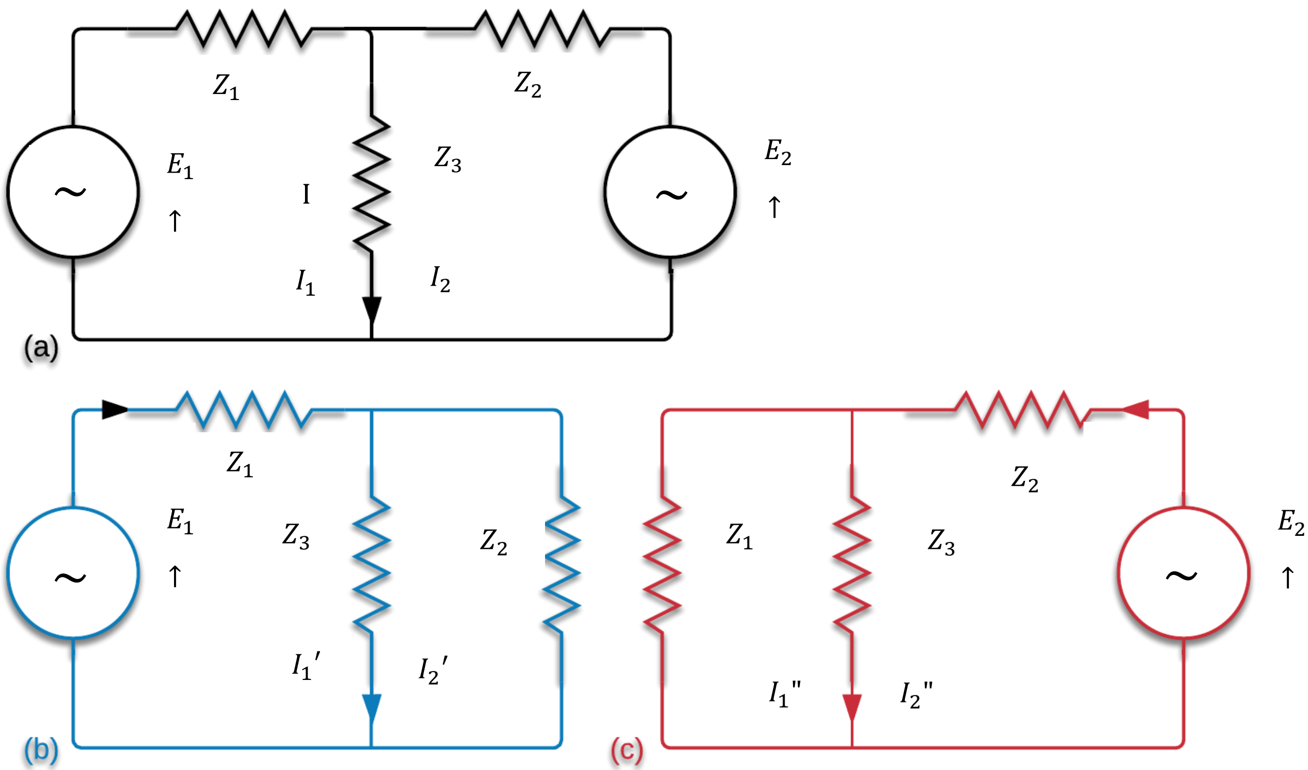


Fig. (a) Two mesh network. (b) Two mesh network when E_1, is removed. (c) Two mesh network when E_1 is removed.

Consider the two voltage sourcesE_1, and E_2 and three impedances Z_1, Z_2 and Z_3 as shown in fig. Let I_1 and I_2 be the two mesh currents then the mesh equations can be written as

$$E_1 = (Z_1 + Z_3)I_1 + Z_3 I_2 \tag{1}$$

$$E_2 = Z_3 I_1 + (Z_2 + Z_3)I_2 \tag{2}$$

From equation (2), we have

$$E_2 - (Z_2 + Z_3)I_2 = Z_3 I_1$$

$$I_1 = \frac{E_2}{Z_3} - \frac{(Z_2 + Z_3)I_2}{Z_3}$$

Substituting the value of I_1 in equation (1), we have

$$E_1 = (Z_1 + Z_3)\left[\frac{E_2}{Z_3} - \frac{(Z_2 + Z_3)I_2}{Z_3}\right] + Z_3 I_2$$

$$E_1 = \frac{(Z_1 + Z_3)E_2}{Z_3} - \frac{(Z_1 + Z_3)(Z_2 + Z_3)I_2}{Z_3} + Z_3 I_2$$

$$E_1 = \frac{(Z_1 + Z_3)E_2}{Z_3} - \frac{(Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)I_2}{Z_3}$$

$$\frac{(Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3)I_2}{Z_3} = \frac{(Z_1 + Z_3)E_2}{Z_3} - E_1$$

So,

$$I_2 = \frac{(Z_1 + Z_3)E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} - \frac{E_1 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \qquad \text{(a)}$$

Substituting the value of I_2 in equation (1), and solving it we get the value of I_1, which is given as

$$E_1 = (Z_1 + Z_3)I_1 + Z_3\left[\frac{(Z_1 + Z_3)E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} - \frac{E_1 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}\right]$$

$$E_1 = (Z_1 + Z_3)I_1 + \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} - \frac{E_1 {Z_3}^2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$(Z_1 + Z_3)I_1 = E_1 + \frac{E_1 {Z_3}^2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} - \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$= E_1\left\{1 + \frac{{Z_3}^2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}\right\} - \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$= E_1\left\{\frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + {Z_3}^2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}\right\} - \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$= E_1\left\{\frac{Z_1(Z_2 + Z_3) + Z_3(Z_2 + Z_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}\right\} - \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$= E_1\left\{\frac{(Z_2 + Z_3)(Z_1 + Z_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}\right\} - \frac{(Z_1 + Z_3)E_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

So

$$(Z_1 + Z_3)I_1 = E_1\left\{\frac{(Z_2 + Z_3)(Z_1 + Z_3)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}\right\} - \frac{(Z_1 + Z_3)E_2Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_1 = \frac{E_1(Z_2 + Z_3)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} - \frac{E_2Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \tag{b}$$

Now considering the circuit of fig. (b), we have the mesh equations

$$E_1 = (Z_1 + Z_3)I_1' + Z_3I_2' \tag{3}$$

$$0 = Z_3I_1' + (Z_2 + Z_3)I_2' \tag{4}$$

Solving equations (3) and (4) we have

$$I_2' = \cancel{\frac{(Z_1 + Z_3)E_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} - \frac{E_1Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

$$I_2' = -\frac{E_1Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \tag{c}$$

And

$$I_1' = \frac{E_1(Z_2 + Z_3)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} - \cancel{\frac{E_2Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}}$$

$$I_1' = \frac{E_1(Z_2 + Z_3)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \tag{d}$$

Referring to circuit of fig. (c), the mesh equations are

$$0 = (Z_1 + Z_3)I_1" + Z_3I_2" \tag{5}$$

$$E_2 = Z_3I_1" + (Z_2 + Z_3)I_2" \tag{6}$$

Solving these equations, we have

$$I_2" = \frac{(Z_1 + Z_3)E_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \cancel{- \frac{E_1Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} \tag{e}$$

$$I_1" = \cancel{\frac{E_1(Z_2 + Z_3)}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}} - \frac{E_2Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \tag{f}$$

From equations (a), (b), (c), (d), (e) and (f), we have

$$I_1 = I_1' + I_1"$$

And

$$I_2 = I_2' + I_2"$$

This proves the theorem.

General Case: *In any linear network containing linear impedances and several sources, the voltage across or the current through any impedance may be calculated by adding algebraically all the individual voltages or currents caused by each source acting alone with all other voltage sources replaced by short circuits and all other current sources replaced by open circuits. By the term, 'linear network' we mean that current in all branches is directly proportional to the driving voltage or e.m.f. impressed.*

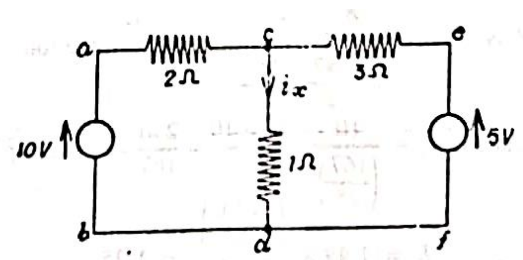
Advantages:

(1) This theorem permits the solution of networks without setting up a large number of simultaneous equations because at a time one generator is used.

(2) If the new generators are introduced into the system, it is not necessary to solve the network from beginning provided the internal impedances of the generators are zero.

(3) If voltages of different frequencies are introduced, this theorem permits a solution to be obtained for each individual frequency. As these solutions are independent of each other, the currents of each frequency flow as if the other frequencies were absent.

Problem 1: Use the superposition r theorem to find the current i_x, in branch cd in the following circuit.

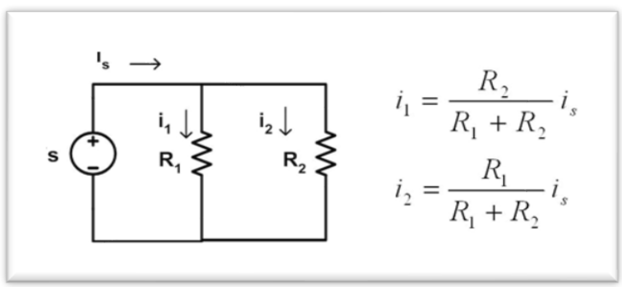


Solution: According to the superposition theorem first we shall calculate the current due to voltage source 10V by short circuiting the 5V source. Then branches *cd* and *ce* will be parallel to each other and in series with *ac* branch; so total resistance is given as

$$R_{T_1} = \frac{1 \times 3}{1+3} + 2 = \frac{3}{4} + 2 = \frac{11}{4}\ \Omega$$

So, the current generated by 10V, source is

$$I_1 = \frac{10}{\frac{11}{4}} = \frac{40}{11}\ amp$$



Hence

$$I_{x_1} = \frac{40}{11} \times \frac{3}{1+3} = \frac{40}{11} \times \frac{3}{4} = \frac{30}{11}\ amp$$

Nose we shall calculate the current due to 5V source by short circuiting 10V source. We see that branches cd and ac are parallel with each other and are in series with ce branch.

So,

$$R_{T_2} = \frac{1 \times 2}{1+2} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\ \Omega$$

So the current generated by 5V source is

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$$I_2 = \frac{5}{\frac{11}{3}} = \frac{15}{11}\ amp$$

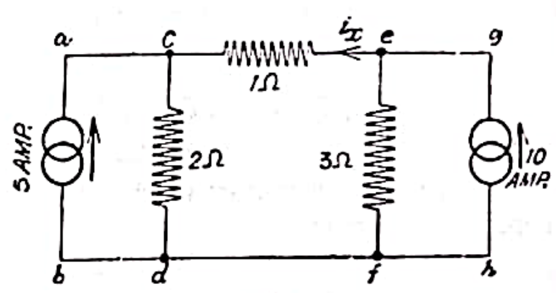
Hence,

$$I_{x_2} = \frac{15}{11} \times \frac{2}{1+2} = \frac{15}{11} \times \frac{2}{3} = \frac{10}{11}\ amp$$

So, total branch current

$$i_x = I_{x_1} + I_{x_2} = \frac{30}{11} + \frac{10}{11} = \frac{40}{11} = \mathbf{3.64\ amp}$$

Problem 2: Use the superposition theorem to find the current i_x in branch *ce* in the circuit shown in fig.



Solution: The same procedure will be adopted here for calculating i_x as we have applied in first problem with a with a difference that here we open circuit the current sources.

First open circuit 10 amp. source, then branches *ce* and *ef* will be in series with each other and in parallel with branch *cd*. So current 5 amp. will be divided at node *c* in *cd* and at *e* in *ef* branches. Applying the current division law at *c*, we have

$$i_{ce} = -i_{x_1} = \frac{5 \times 2}{2+1+3} = \frac{10}{6}\ amp$$

$$i_{x_1} = -\frac{10}{6}\ amp$$

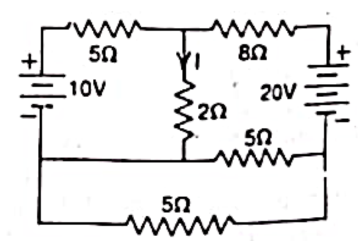
Now open circuit 5 amp. source and calculate current i_{x_2} due to 10 amp. source. We apply the current division law at *e*, we have

$$i_{x_2} = \frac{10 \times 3}{2+1+3} = \frac{30}{6}\ amp$$

So, total branch current

$$i_x = i_{x_1} + i_{x_2} = -\frac{10}{6} + \frac{30}{6} = \frac{20}{6} = \mathbf{3.33\ amp}$$

Problem 3: Apply superposition theorem for finding current through 2Ω resistance of the circuit shown in fig.



Solution: Considering only 10V source, the equivalent circuit is shown in fig. (a). Here lower 5Ω resistances are in parallel. The equivalent resistance is $\frac{5}{2}\Omega$. This $\frac{5}{2}\Omega$ resistance is in series with 8Ω resistance. So, the equivalent resistance is $\frac{21}{2}\Omega$. Further 2Ω resistance and $\frac{21}{2}\Omega$ resistance are in parallel. Hence the resistance of the portion of this circuit is given by

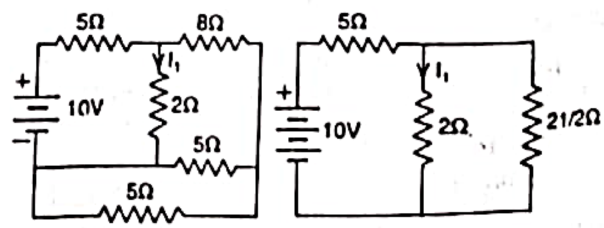


Fig. (a)

$$\frac{1}{R'} = \frac{1}{2} + \frac{2}{21} = \frac{21+4}{42} = \frac{25}{42}$$

$$R' = \frac{42}{25}\Omega$$

R' is in series of 5Ω resistance and hence the total resistance of the circuit is given by

$$5 + \frac{42}{25} = \frac{167}{25}\ \Omega$$

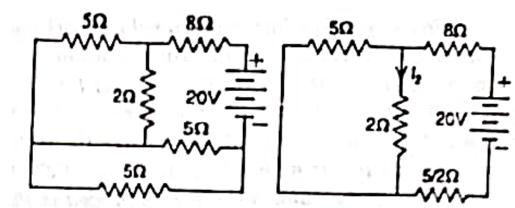


Fig. (b)

Current

$$I_1{}' = \frac{10}{\frac{167}{25}} = \frac{250}{167} = 1.49\ amp$$

And branch current due to 10V source

$$I_1 = 1.49 \times \frac{\left(\frac{21}{2}\right)}{\left(\frac{21}{2} + 2\right)} = 1.25\ amp$$

Similarly, for 20V source alone, the circuit is shown in fig. (b). The equivalent resistance of the circuit is given by

$$\frac{5}{2} + \frac{5 \times 2}{5+2} + 8$$

$$= 8 + \frac{10}{7} + \frac{5}{2} = 11.93\ \Omega$$

Let $I_2{}'$, be the current flowing through this equivalent resistance. Then,

$$I_2{}' = \frac{20}{11.93} = 1.68\ amp$$

Further current through 2Ω resistor due to 20V source

$$I_2 = 1.68 \times \frac{5}{5+2} = 1.20\ amp$$

So, total branch current

$$I = I_1 + I_2 = 1.25 + 1.20 = \mathbf{2.45\ amp}$$

Internal Impedance of a Source:

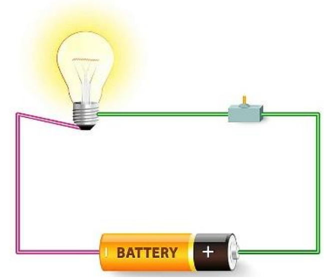


Fig: 1(a)

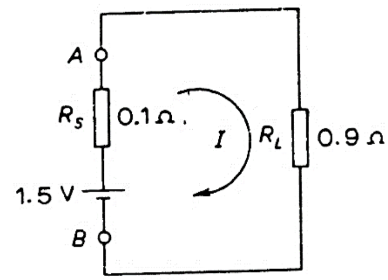


Fig: 1(b)

All electrical energy sources have some internal impedance (or resistance). It is due to this internal impedance that the source does not behave ideally. When a voltage source supplies power to a load, its terminal voltage *(voltage available at its terminals)* drops. A cell used in a torch has a voltage of 1.5 V across its two electrodes when nothing is connected to it. However, when connected to a bulb, its voltage becomes less than 1.5 V. Such a reduction in the terminal voltage of the cell may be explained as follows. Figure 1(a) shows a cell of 1.5 V connected to a bulb. When we say "cell of 1.5 V", we mean a cell whose open-circuit voltage is 1.5 V. In the equivalent circuit of Fig. 1(b), the bulb is replaced by a load resistor R_L, (of, say, 0.9 Ω), and the cell is replaced by a constant voltage source of 1.5 V in series with the internal resistance R_s (of, say, 0.1 Ω. The total resistance in the circuit is now 0.1 + 0.9 = 1.0 Ω. Since the net voltage that sends current into the circuit is 1.5 V, the current in the circuit is

$$I = \frac{V}{R} = \frac{1.5}{1.0} = 1.5\ A$$

The terminal voltage (the voltage across the terminals AB) of the cell is same as the voltage across the load resistor R_L. Therefore,

$$V_{AB} = I \times R_L = 1.5 \times 0.9 = 1.35\ V$$

The voltage that drops because of the internal resistance is

$$= 1.5 - 1.35 = 0.15\ V$$

Note this, if the internal resistance of the cell were smaller (compared to the load resistance), voltage drop would also have been smaller than 0.15 V. The internal resistance (or impedance in case case of ac source) of a source may be due to one or more of the following reasons:

(i) The resistance of the electrolyte between the electrodes, in case of a cell.

(ii) The resistance of the armature winding in case of an alternator or a dc generator.

(iii) The output impedance of the active device like a transistor or vacuum tube in case of an oscillator (or signal generator), and rectification-type dc supply.

Concept of Voltage Source

Consider an ac source. Let V_S be its open-circuit voltage (i.e., the voltage which exists across its terminals when nothing is connected to it), and Z_S be its internal impedance. Let it be connected to a load impedance Z_L, whose value can be varied, as shown in Fig. 1.

Now, suppose Z_L, is infinite. It means that the terminals AB of the source are open-circuited. Under this condition, no current can flow. The terminal voltage V_T is obviously the same as the emf Vs, since there is no voltage drop across Z_S. Let us now connect a finite variable load impedance Z_L, and then go on reducing its value. As we do this, the current in the circuit goes on increasing. The voltage drop across Z_S also goes on increasing. As a result, the terminal voltage V_T goes on decreasing.

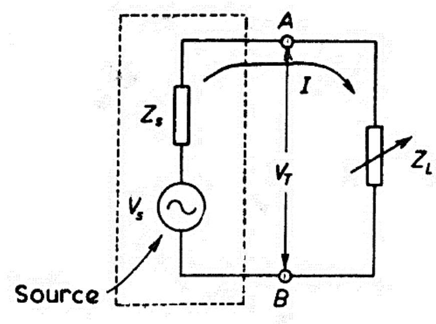


Fig. 1: A variable load connected to an ac source

For a given value of Z_L, the current in the circuit is given as

$$I = \frac{V_S}{Z_S + Z_L}$$

Therefore, the terminal voltage of the source, which is the same as the voltage across the load, is

$$V_T = I \times Z_L = \frac{V_S}{Z_S + Z_L} \times Z_L = \frac{V_S}{1 + \frac{Z_S}{Z_L}} \qquad (1)$$

From the above equation, we find that if the ratio $\frac{Z_S}{Z_L}$ is small compared to unity, the terminal voltage V_T remains almost the same as the voltage V_S. Under this condition, the source behaves as a good voltage source. Even if the load impedance changes, the terminal voltage of the source remains practically constant (provided the ratio $\frac{Z_S}{Z_L}$ is quite small). Such a source can then be said to be a "good (hut not ideal) voltage source".

Ideal Voltage Source

It would have been ideal, if the terminal voltage of a source remains fixed whatever be the load connected to it. In other words, a voltage source should ideally provide a fixed terminal voltage even though the current drain (or load resistance) may vary. In Eq. 1, to make the terminal voltage fixed for any value of Z_L, the only way is to make the internal impedance Z_S zero. Thus, we infer that an ideal voltage source must have zero internal impedance. The symbolic representation of dc and ac ideal voltage sources are given in Fig. 1. And Figure 2 gives the characteristics of an ideal voltage source. The terminal voltage V_T is seen to be constant at V_S for all values of load current (load current varies as the load impedance is changed).

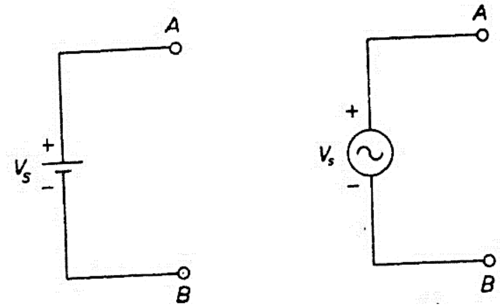


Fig. 1: Symbolic representation of an ideal voltage source: (a) DC voltage source (b) AC voltage source

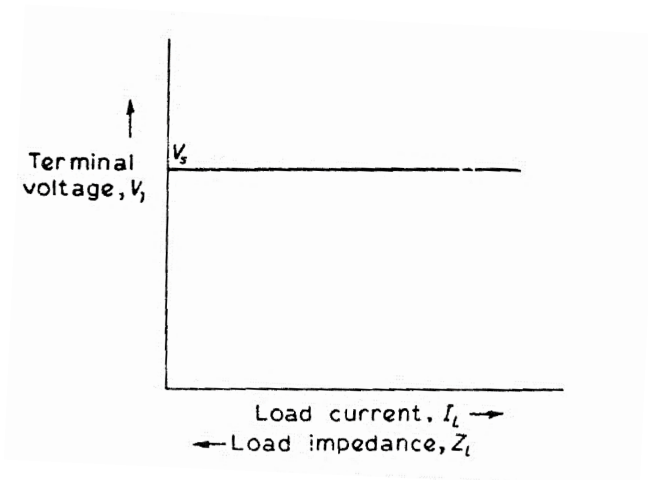


Fig. 2: V-I characteristics of an ideal voltage source.

Practical Voltage Source

An ideal voltage source is not practically possible. There is no source which can attain it terminal voltage constant when its terminals are short-circuited. If it could do so, it would mean that it can supply an infinite amount of power to a short-circuit. This is not possible. Hence, an ideal voltage source does not exist in practice. However, the concept of an ideal voltage source is very helpful in understanding the circuits containing a practical voltage source.

A practical voltage source can be considered to consist of an ideal voltage source in series with an impedance. This impedance is called the internal impedance of the source. The symbolic representation of practical voltage sources are shown in Fig. 1.

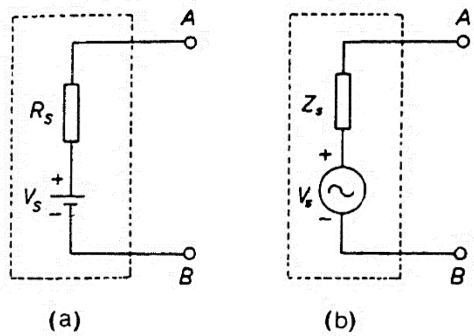


Fig. 1: Practical voltage source: (a) DC voltage source (b) AC voltage source

It is not possible to reach any other terminal except A and B. These are the terminals available for making external connections. In the dc source, since the upper terminal of the ideal voltage source is marked positive, the terminal A will be positive with respect to terminal B. In the ac source in Fig.1(b), the upper terminal of the ideal voltage source is marked as positive and lower as negative. The marking of positive and negative on an ac source does not mean the same thing as the markings on a dc source. Here (in ac), it means that the upper terminal (terminal A) of the ideal voltage source is positive with respect to the lower terminal at that particular instant. In the next half-cycle of ac, the lower terminal will be positive and the upper negative. Thus, the positive and negative markings on an ac source indicate the polarities at a given instant of time. In some books you will find the reference polarities marked by, instead of positive and negative signs, an arrow pointing towards the positive terminal.

The question naturally arises: What should be the characteristics of a source so that it may be considered a good enough constant voltage source? An ideal voltage source, of course, must have zero internal impedance. In practice, no source can be an ideal one. Therefore, it is necessary to determine how much the value of the internal impedance Z_S should be, so that it can be called a good practical voltage source.

Let us consider an example. A dc source has an open-circuit voltage of 2 V, and internal resistance of only 1 Ω. It is connected to a load resistance R_L as shown in Fig. 2(a). The load resistance can assume any value ranging from 1 Ω to 10 Ω. Let us now find the variation in the terminal voltage of the source. When the load resistance R_L is 1 Ω the total resistance in the circuit is 1 Ω + 1 Ω = 2 Ω. The current in the circuit is

$$I_T = \frac{V_S}{R_S + R_{L_1}} = \frac{2}{1+1} = 1\ A$$

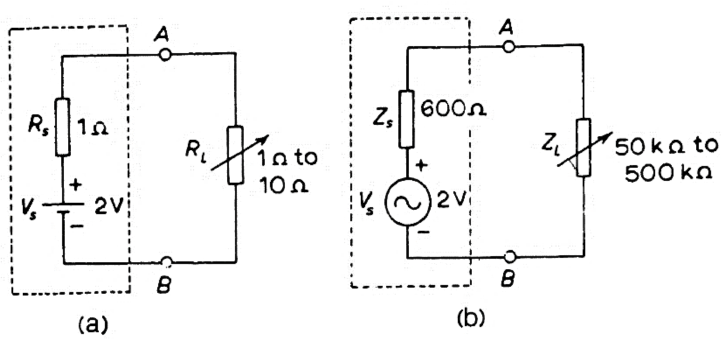


Fig. 2: Voltage sources connected to variable loads

The terminal voltage is then

$$V_{T_1} = I_1 \times R_{L_1} = \frac{V_S}{R_S + R_{L_1}} \times R_{L_1}$$

$$= \frac{2}{1+1} \times 1 = 1.0\,V$$

When the load resistance becomes 10 Ω, the total resistance in the circuit becomes 10 Ω + 1 Ω =11 Ω. We can again find the terminal voltage as

$$V_{T_2} = I_2 \times R_{L_2} = \frac{V_S}{R_S + R_{L_2}} \times R_{L_2}$$

$$= \frac{2}{1+10} \times 10 = 1.818\,V$$

Thus, we find that the maximum voltage available across the terminals of the source is 1.818 V. When the load resistance varies between its extreme limits—from 1 Ω to 10 Ω, the terminal voltage varies from 1 V to 1.818 V. This is certainly a large variation. The variation in the terminal voltage is more than **40 %** of the maximum voltage.

Let us consider another example. A 600 Ω, 2 V ac source is connected to a variable load, as shown in Fig. 2(b). The load impedance Z_L can vary from 50 KΩ to 500 KΩ, again a variation having the same ratio of 1 : 10, as in the case of the first example. We can find the variation in the terminal voltage of the source. When the load impedance is 50 KΩ, the terminal voltage is

$$V_{T_1} = I_1 \times Z_{L_1} = \frac{V_S}{Z_S + Z_{L_1}} \times Z_{L_1}$$

$$= \frac{2}{600 + 50000} \times 50000 = 1.976\,V$$

When the load impedance is 500 KΩ the terminal voltage is

$$V_{T_2} = I_2 \times Z_{L_2} = \frac{V_S}{Z_S + Z_{L_2}} \times Z_{L_2}$$

$$= \frac{2}{600 + 500000} \times 500000 = 1.997\,V$$

With respect to the maximum value, the percentage variation in terminal voltage

$$= \frac{1.997 - 1.976}{1.997} \times 100 = 1.05\ \%$$

We can now compare the two examples. In the first case, although the internal resistance of the dc source is only 1Ω, yet it is not justified to call it a constant voltage source. Its terminal voltage varies by more than 40 %. In the second case, although the internal impedance of the ac source is 600 Ω, it may still be called a practical constant voltage source, since the variation in its terminal voltage is quite small (only 1.05 %). Thus, we conclude that it is not the absolute value of the internal impedance that decides whether a source is a good constant voltage source or not. It is the value of the internal impedance relative to the load impedance that is important. The lesser the ratio

Z_S/Z_L (in the first example, this ratio varies from 1 to 0.1, whereas in the second example it varies from 0.012 to 0.0012), the better is the source as a constant voltage source.

No practical voltage source can be an ideal voltage source. Thus, no practical voltage source can have the V-I characteristic as shown in Fig. before. When the load current increases, the terminal voltage of a practical voltage source decreases. The characteristic is then modified to that shown in Fig. 3(a). It is sometimes preferred to take voltage on the x-axis and current on the y-axis. The V-I characteristic of a practical voltage source then looks like the one shown in Fig. 3(b).

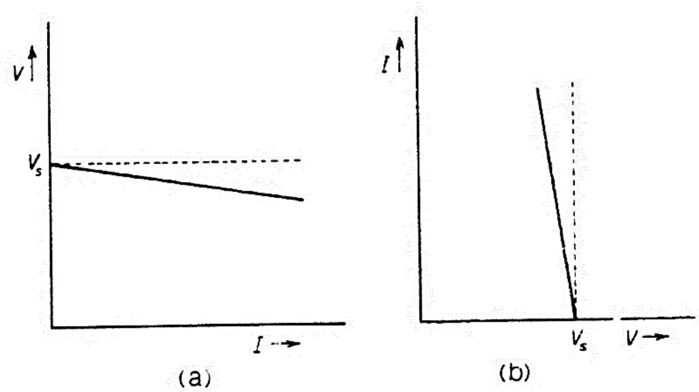


Fig. 3: Two ways of drawing VI characteristics of a practical voltage source

Concept of Current Source

Like a constant voltage source, there may be a constant current source – a source that supplies a constant current to a load even its impedance varies. Ideally, the current supplied by it should remain constant, no matter what the load impedance is.

A symbolic representation of such an ideal current source is shown in Fig. 1(a). The arrow inside the circle indicates the direction in which current will flow in the circuit when a load is connected to the source. Fig. 1(b) shows the V-I characteristic of an ideal current source. Let us connect a variable load impedance Z_L to a constant current source as shown in Fig. 1(c). As stated above, the current supplied by the source should remain constant at is for all values of load impedance.

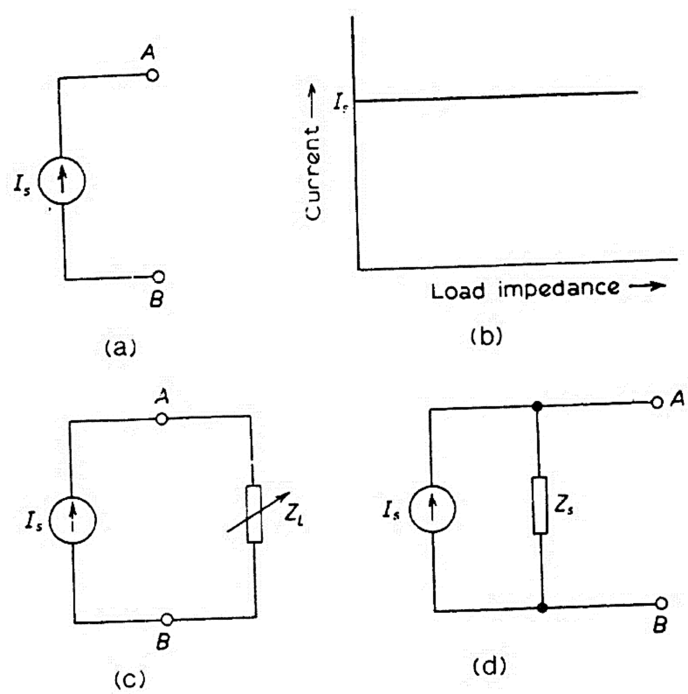


Fig.1: (a) Symbol for an ideal current source (b) V-I characteristic of an ideal current source (c) A variable load connected to an ideal current source (d) Symbol for a practical current source

It means even if Z_L is made infinity, the current through this should remain I_S (same). Now, we must see if any practical current source could satisfy this condition. The load impedance $Z_L = \infty$ means no conducting path, external to the source, exists between the terminals A and B. Hence, it is a physical impossibility for current to flow between terminals A and B. If the source could maintain a current Is through an infinitely large load impedance, there would have been an infinitely large voltage drop across the load. It would then have consumed infinite power from the source. Of course, no practical source could ever supply infinite power.

The maximum voltage that the current source can deliver to the load is called **compliance voltage.** During the variation in the load the current source work like ideal source, provides the unlimited resistance but, when the voltage value at the output reaches to compliance voltage, then it starts to behave like a real source and provides the limited value of resistance.

A practical current source supplies current I_S to a short-circuit (i.e. when Z_L= 0). That is why the current I_S is called short-circuit current. But, when we increase the load impedance, the current falls below I_S. When the load impedance Z_L is made infinite (i.e., the terminals A and B are open-circuited), the load current reduces to zero. It means there should be some path (inside the source itself) through which the current I_S can flow. When some finite load impedance is connected, only a part of this current I_S flows through the load. The remaining current goes through the path inside the source. This inside path has an impedance Z_S, and is called the internal impedance. The symbolic representation of such a practical current source is shown in Fig. 1(d).

Now, if terminals AB are open-circuited ($Z_L = \infty$) in Fig. 1(d), the terminal voltage does not have to be infinite. It is now a finite value, $V_T = I_S Z_S$. It means that the source does not have to supply infinite power!

Practical Current Source

An ideal current source is merely an idea. In practice, an ideal current source cannot exist. Obviously, there cannot be a source that can supply constant current even if its terminals are open-circuited. The reason why an actual source does not work as an ideal current source is that its internal impedance is not infinite. A practical current source is represented by the symbol shown in Fig. 1(d) in previous section. The source impedance Z_S is put in parallel with the ideal current source I_S. Now, if we connect a load across the terminals A and B, the load current will be different from the current I_S. The current I_S now divides itself between two branches—one made of the source impedance Z_S inside the source itself, and the other made of the load impedance Z_L external to the source.

Let us find the conditions under which a source can work as a good (practical) current source. in Fig. 1(a), a load impedance Z_L is connected to a current source. Let I_S be the short-circuit current of the source, and Z_S be its internal impedance. The current I_S is seen to be divided into two parts—I_1 through Z_S and I_L through Z_L. That is,

$$I_S = I_1 + I_L$$

$$I_1 = I_S - I_L$$

Since the impedance Z_S and Z_L are in parallel, the voltage drop across each should be equal, i.e.,

$$I_1 Z_S = I_L Z_L$$

$$(I_S - I_L) Z_S = I_L Z_L$$

$$I_L = \frac{I_S Z_S}{Z_S + Z_L}$$

$$I_L = \frac{I_S}{1 + \frac{Z_L}{Z_S}} \qquad (1)$$

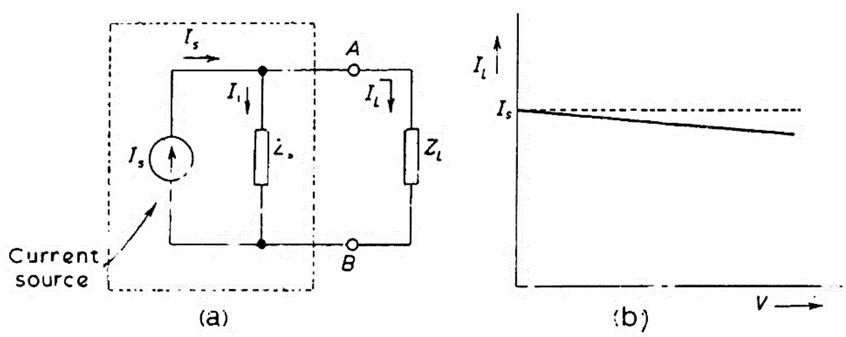


Fig. 1: (a) Practical current source feeding current to a load impedance (b) V-I characteristic of a practical current source

This equation tells us that the load current IL will remain almost the same as the current I_S, provided the ratio Z_L/Z_S is small compared to unity. The source then behaves as a good current source. In other words, the larger the value of internal impedance Z_S (compared to the load impedance Z_L), the smaller is the ratio Z_L/Z_S, and the better it works as a constant current source.

From Eq.1, we see that the current $I_L = I_S$, when $Z_L = 0$. But, as the value of load impedance is increased, the current I_L is reduced. For a given increase in load impedance Z_L, the corresponding reduction in load current I_L is much smaller. Thus, with the increase in load impedance, the terminal voltage ($V = I_L Z_L$) also increases. The V-I characteristic of a practical current source is shown in Fig. 1(b).

$$V_T = \frac{V_S}{1 + \frac{Z_S}{Z_L}}$$

$$I_L = \frac{I_S}{1 + \frac{Z_L}{Z_S}}$$

If,

$$Z_S = 50\Omega$$

$$Z_{L_1} = 1\Omega \quad \text{in this case, the source acts more likely as a voltage source}$$

$$Z_{L_2} = 1000\Omega \quad \text{in this case, the source acts more likely as a current source}$$

Equivalence Between Voltage Source and Current Source

Practically, a voltage source is not different from a current source. In fact, a source can either work as a current source or as a voltage source. It merely depends upon its working conditions. If the value of the load impedance is very large compared to the internal impedance of the source, it proves advantageous to treat the source as a voltage source. On the other hand, if the value of the load impedance is very small compared to the internal impedance, it is better to represent the source as a current source. From the circuit point of view, it does not matter at all whether the source is treated as a current source or a voltage source. In fact, it is possible to convert a voltage source into a current source and vice-versa.

Conversion of Voltage Source into Current Source and vice versa

Consider an ac source connected to a load impedance Z_L. The source can either be treated as a voltage source or a current source, as shown in Fig.1. The voltage-source representation consists of an ideal voltage source V_S in series with a source impedance Z_{S_1}. And the current-source representation consists of an ideal current source I_S in parallel with source impedance Z_{S_2}. These are the two representations of the same source. Both types of representations must appear the same to the externally connected load impedance Z_L. They, must give the same results.

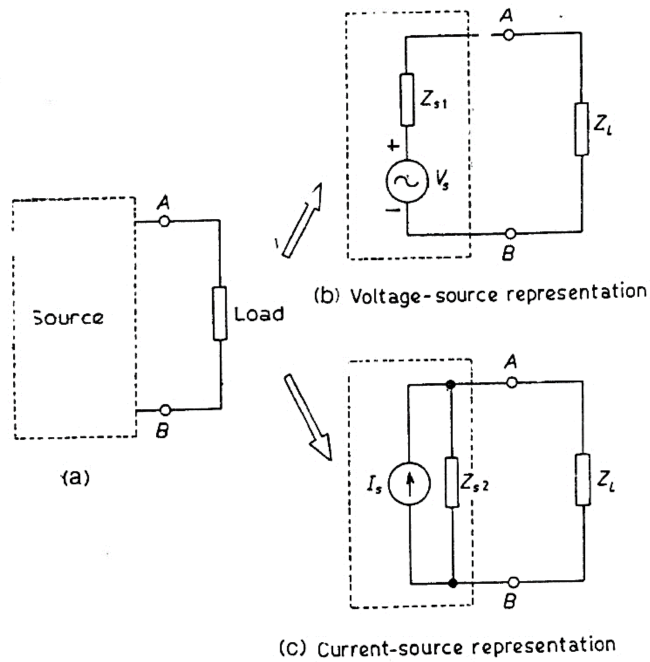


Fig.1: A source connected to a load

In Fig.1(b), if the load impedance Z_L is reduced to zero (i.e., the terminals A and B are short-circuited), the current through this short is given as

$$I_L(short - circuit) = \frac{V_S}{Z_{S_1}} \qquad (1)$$

We want both the representations (voltage-source and current-source) to give the same results. This means that current source in Fig.1(c) must also give the same current (as given by Eq.1) when terminals A and B are shorted. But the current obtained by shorting the terminals A and B of Fig.1(c) is simply the source current I_S (the source impedance Z_{S_2} connected in parallel with a short-circuit is as good as not being present). Therefore, we conclude that the current I_S of the equivalent current source must be the same as that given by Eq.1. Thus

$$I_L(short - circuit) = I_S = \frac{V_S}{Z_{S_1}} \qquad (2)$$

Again, the two representations of the source must give the same terminal voltage when the load impedance Z_L is disconnected from the source (i.e., when the terminals A and B are open-circuited. In Fig.1(b), the open-circuit terminal voltage is simply V_S. There is no voltage drop across the internal impedance Z_{S_1}. Let us find out the open-circuit voltage in the current-source representation of Fig. 1(c). When the terminals A and B are open-circuited, the whole of the current is flows through the impedance Z_{S_2}. The terminal voltage is then the voltage drop across this impedance. That is

$$V_T\ (open - circuit) = I_S Z_{S_2} \qquad (3)$$

Therefore, if the two representations of the source are to be equivalent, we must have

$$V_T = V_S$$

Using Eqs. 2 and 3, we get

$$I_S Z_{S_1} = I_S Z_{S_2}$$

$$Z_{S_1} = Z_{S_2} = Z_S \qquad (say)$$

Then both Eqs. 2 and 3 reduce to

(4)

$$V_S = I_S Z_S$$

It may be noted (see Eq. 3) that in both the representations of the source, the source impedance as faced by the load impedance at the terminals AB, is the same (impedance Z_S). Thus, we have established the equivalence between the voltage-source representation and current-source representation of Fig.1, for short-circuits and for open-circuits. But we are not sure that the equivalence is valid for any other value of load impedance. To test this, let us check whether a given impedance Z_L, draws the same amount of current when connected either to the voltage-source representation or to the current-source representation.

In Fig. 1(b), the current through the load impedance is

$$I_{L_1} = \frac{V_S}{Z_S + Z_L} \tag{5}$$

In Fig. 1(c), the current I_S divides into two branches. Since the current divides itself into two branches in inverse proportion of the impedances, the current through the load impedance Z_L is

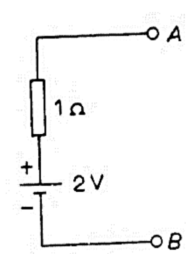
$$I_{L_2} = I_S \times \frac{Z_S}{Z_S + Z_L} = \frac{I_S Z_S}{Z_S + Z_L}$$

By making use of Eq. 4, the above equation can be written as

$$I_{L_2} = \frac{V_S}{Z_S + Z_L} \tag{6}$$

We now see that the two currents I_{L_1} and I_{L_2} as given by Eqs. 5 and 6 are exactly the same. Thus, the equivalence between the voltage-source and current-source representations of Fig.1 is completely established. We may convert a given voltage source into its equivalent current source by using Eq.4. Similarly, any current source may be converted into its equivalent voltage source by using the same equation.

Example 1: Figure shows a dc voltage source having an open circuit voltage of 2 V and an internal impedance of 1Ω. Obtain its equivalent current-source representation.



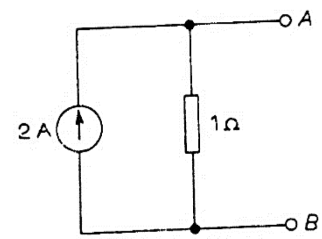
Solution:

If we short-circuit the terminals A and B of the voltage source, the current supplied by the source is

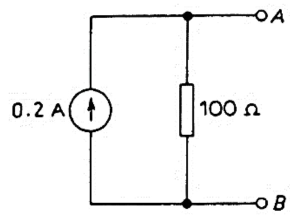
$$I(short - circuit) = \frac{V_S}{R_S}$$

$$= \frac{2}{1} = 2\ A$$

In the equivalent current-source representation, the current source is of 2 A. The source impedance of 1Ω is connected in parallel with this current source. The equivalent current source obtained is shown in Fig. next.



Example 2: Obtain an equivalent voltage source of the ac current source shown in Fig.

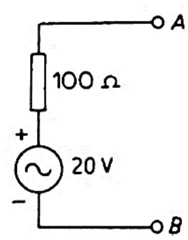


Solution: The open-circuit voltage across terminals A and B is given as

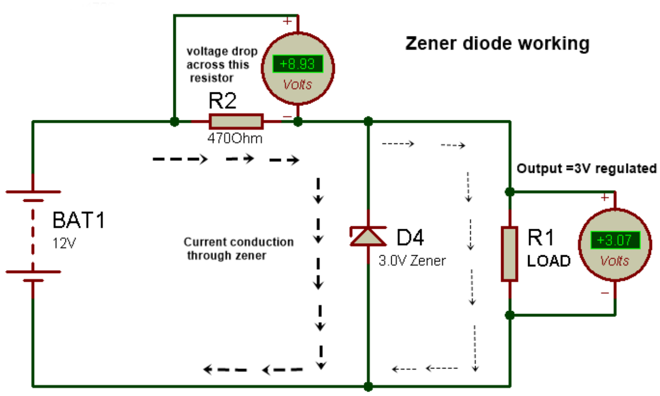
$$V_S(open - circuit) = I_S Z_S$$

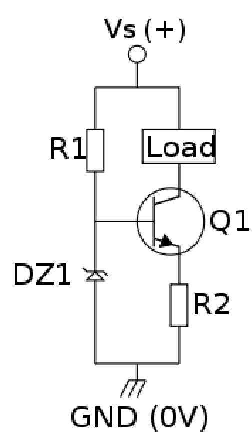
$$= 0.2 \times 100 = 20\ V$$

This will be the value of the "ideal voltage source" in the equivalent voltage- source representation. The source impedance Z_S is put in series with the ideal voltage source. Thus, the equivalent voltage-source representation of the given current source is as



Examples of practical current and voltage source





$$I_{R2}(= I_E = I_C) = \frac{V_{R2}}{R_{R2}} = \frac{V_Z - V_{BE}}{R_{R2}}$$

Reduction of Complicated Networks-Equivalent Star and Delta Circuits and their Conversions

Delta Star Conversion Theorem:

The three elements of a network may he arranged either as $T-section$ or a $\pi-section$ as shown in fig. (1a) and (1b) respectively. Further, the T-section may be redrawn as a $star$ or $Y-section$ (fig. 1c) and $\pi-section$ as a mesh or delta $\Delta-section$ (fig. 1d).

According to delta star conversion theorems, a $T-section$ or $star\ section$ can be interchanged to a $\pi-section$ or $\Delta-section$ and vice versa at any frequency, provided that, certain relations are maintained between the elements of two sections.

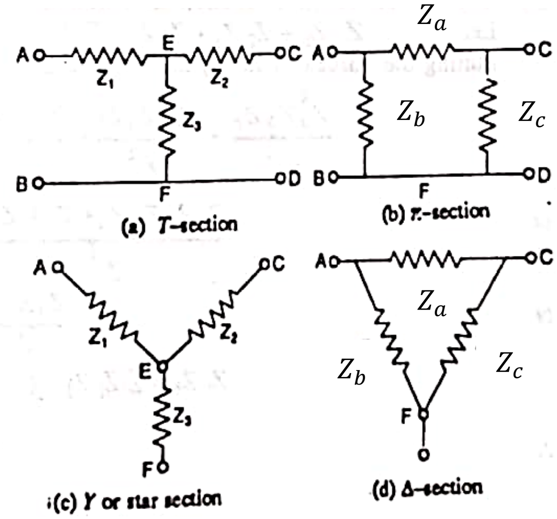


Fig. 1

Conversion from $\pi-section$ to $T-section$

Equating the impedances looking between terminals A and C, (Fig.1 c and d), we have

$$Z_{AC}=Z_1+Z_2 \qquad \text{(Fig.1c)}$$

$$\frac{1}{Z_{AC}}=\frac{1}{Z_a}+\frac{1}{Z_b+Z_c} \qquad \text{(Fig.1d)}$$

$$=\frac{(Z_b+Z_c)+Z_a}{Z_a(Z_b+Z_c)}$$

$$Z_{AC}=\frac{Z_a(Z_b+Z_c)}{Z_a+Z_b+Z_c}$$

So, we can write from above equations

$$Z_1+Z_2=\frac{Z_a(Z_b+Z_c)}{Z_a+Z_b+Z_c}=\frac{Z_a(Z_b+Z_c)}{Z} \qquad (1)$$

Now equating the impedances looking between terminals C and F, (Fig.1 c and d), we have

$$Z_{CF}=Z_2+Z_3 \qquad \text{(Fig.1c)}$$

$$\frac{1}{Z_{CF}}=\frac{1}{Z_c}+\frac{1}{Z_a+Z_b} \qquad \text{(Fig.1d)}$$

$$Z_{CF}=\frac{Z_c(Z_a+Z_b)}{Z_a+Z_b+Z_c}=\frac{Z_c(Z_a+Z_b)}{Z}$$

So, we can write from above equations

$$Z_2+Z_3=\frac{Z_c(Z_a+Z_b)}{Z_a+Z_b+Z_c}=\frac{Z_c(Z_a+Z_b)}{Z} \qquad (2)$$

Similarly, equating the impedances looking between terminals A and F, (Fig.1 c and d), we get

$$Z_{AF} = Z_1 + Z_3$$

$$\frac{1}{Z_{AF}} = \frac{1}{Z_b} + \frac{1}{Z_a + Z_c}$$

$$Z_{AF} = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c}$$

So, we can write from above equations

$$Z_1 + Z_3 = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c} = \frac{Z_b(Z_a + Z_c)}{Z} \qquad (3)$$

where $Z = Z_a + Z_b + Z_c$.

Adding eqs. (1) and (3) and then subtracting eq. (2), we have

$$(Z_1 + Z_2) + (Z_1 + Z_3) - (Z_2 + Z_3) = \frac{Z_aZ_b + Z_aZ_c}{Z} + \frac{Z_bZ_a + Z_bZ_c}{Z} - \frac{Z_cZ_a + Z_cZ_b}{Z}$$

$$2Z_1 = \frac{2Z_aZ_b}{Z}$$

$$Z_1 = \frac{Z_aZ_b}{Z} \qquad (a)$$

Similarly, adding eqs. (1) and (2) and subtracting (3), we get

$$Z_1 + Z_2 = \frac{Z_a(Z_b + Z_c)}{Z}$$

$$Z_2 + Z_3 = \frac{Z_c(Z_a + Z_b)}{Z}$$

$$Z_1 + Z_3 = \frac{Z_b(Z_a + Z_c)}{Z}$$

$$(Z_1 + Z_2) + (Z_2 + Z_3) - (Z_1 + Z_3) = \frac{Z_aZ_b + Z_aZ_c}{Z} + \frac{Z_cZ_a + Z_cZ_b}{Z} - \frac{Z_bZ_a + Z_bZ_c}{Z}$$

$$2Z_2 = \frac{2Z_aZ_c}{Z}$$

$$Z_2 = \frac{Z_aZ_c}{Z} \qquad (b)$$

Finally, adding eqs. (2) and (3) and subtracting (1), we get

$$(Z_2 + Z_3) + (Z_1 + Z_3) - (Z_1 + Z_2) = \frac{Z_cZ_a + Z_cZ_b}{Z} + \frac{Z_bZ_a + Z_bZ_c}{Z} - \frac{Z_aZ_b + Z_aZ_c}{Z}$$

$$2Z_3 = \frac{2Z_bZ_c}{Z}$$

$$Z_3 = \frac{Z_bZ_c}{Z} \qquad (c)$$

Therefore, if impedances of $\pi - section$ are known, then impedances of $T - section$ may be calculated using eqs. (a), (b) and (c). (2)

Conversion from $T - section$ to $\pi - section$

Let $Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \sum Z_1Z_2$

Putting the values of Z_1, Z_2 and Z_3 from eqs. (a), (b) and (c), we get

$$\frac{Z_aZ_b}{Z}\frac{Z_aZ_c}{Z} + \frac{Z_aZ_c}{Z}\frac{Z_bZ_c}{Z} + \frac{Z_bZ_c}{Z}\frac{Z_aZ_b}{Z} = \sum Z_1Z_2$$

$$\frac{Z_a^2Z_bZ_c}{Z^2} + \frac{Z_c^2Z_aZ_b}{Z^2} + \frac{Z_b^2Z_aZ_c}{Z^2} = \sum Z_1Z_2$$

$$\frac{Z_aZ_bZ_c(Z_a + Z_b + Z_c)}{Z^2} = \sum Z_1Z_2$$

$$\frac{Z_aZ_bZ_c}{Z} = \sum Z_1Z_2 \qquad [\because\ Z = Z_a + Z_b + Z_c]$$

$$Z_1Z_c = \sum Z_1Z_2 \qquad [\because\ Z_1 = \tfrac{Z_aZ_b}{Z}]$$

$$Z_c = \frac{\sum Z_1Z_2}{Z_1} \tag{d}$$

Similarly,

$$Z_b = \frac{\sum Z_1Z_2}{Z_2} \tag{e}$$

and

$$Z_a = \frac{\sum Z_1Z_2}{Z_3} \tag{f}$$

Therefore, if the impedances of $T - section$ are known, then impedances of a $\pi - section$ may be calculated using eqs. (d), (e) and (f).

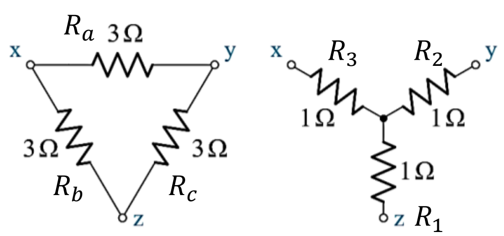
Problem1: 1: Assume we have a Δ circuit with 3 Ω resistors. Derive the Y-equivalent by using the $\Delta \rightarrow Y$ equations.

Solution:

$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc} = \frac{3 \cdot 3}{3+3+3} = 1\,\Omega$$

$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc} = \frac{3 \cdot 3}{3+3+3} = 1\,\Omega$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc} = \frac{3 \cdot 3}{3+3+3} = 1\,\Omega$$



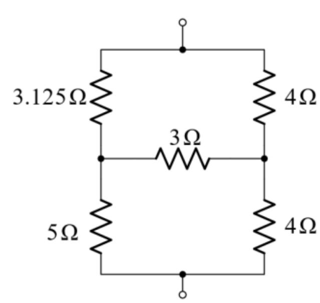
Going in the other direction, from $Y \rightarrow \Delta$, looks like this,

$$Ra = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R1} = \frac{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1}{1} = 3\,\Omega$$

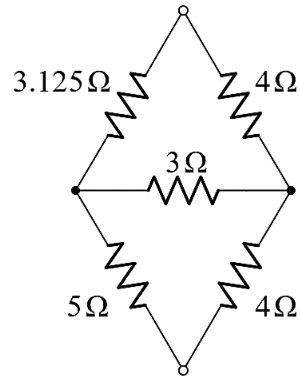
$$Rb = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R2} = \frac{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1}{1} = 3\,\Omega$$

$$Rc = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R3} = \frac{1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1}{1} = 3\,\Omega$$

Example 2: Find the equivalent resistance between the top and bottom terminals.

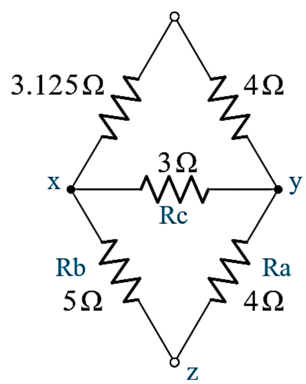


Solution: First, let's redraw the schematic to emphasize we have two Δ connections stacked one on the other.

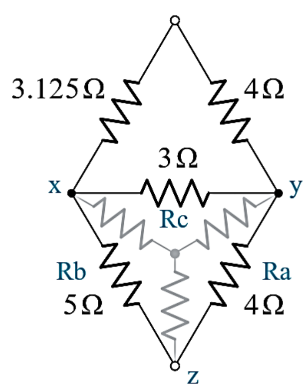


Now select one of the Δ to convert to a Y. We will perform a Δ→Y transformation and see if it breaks the logjam, opening up other opportunities for simplification.

We go to work on the bottom Δ (an arbitrary choice). *Very carefully* label the resistors and nodes. To get the right answers from the transformation equations, it is critical to keep the resistor names and node names straight. *Rc* must connect between nodes *x* and *y*, and so on for the other resistors. Refer to *Diagram 1* above for the labeling convention.



When we perform the transform on the lower Δ, the black Δ resistors will be replaced by the new gray Y resistors, like this:



Perform the transform yourself before looking at the answer. Check that you select the right set of equations. **Compute three new resistor values to convert the Δ to a Y, and draw the complete circuit.**

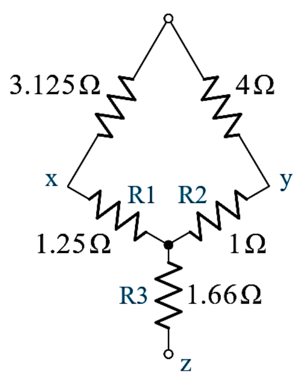
Apply the transformation equations for $\Delta \to \mathrm{Y}$.

$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc} = \frac{5 \cdot 3}{4 + 5 + 3} = \frac{15}{12} = 1.25\,\Omega$$

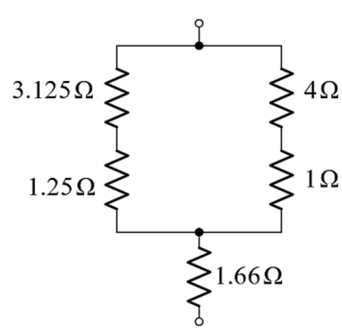
$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc} = \frac{4 \cdot 3}{4 + 5 + 3} = \frac{12}{12} = 1\,\Omega$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc} = \frac{4 \cdot 5}{4 + 5 + 3} = \frac{20}{12} = 1.66\,\Omega$$

Derive an equivalent Y network by substituting the Δ resistors. Make sure the Y resistor names connect between the proper node names. Refer to *Diagram 1* above for the labeling convention.



And voilà! Check out our circuit. It now has series and parallel resistors where it had none before. Continue simplification with series and parallel combinations until we get down to a single resistor between the terminals. Redraw the schematic again to square up the symbols into a familiar style.



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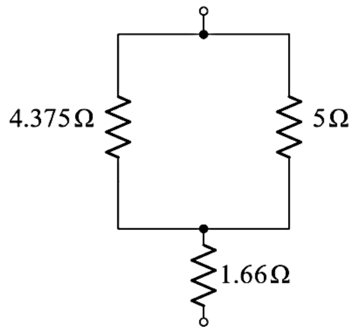
We proceed through the remaining simplification steps just as we did before in the article on Resistor Network Simplification.

On the left branch, $3.125 + 1.25 = 4.375\,\Omega$

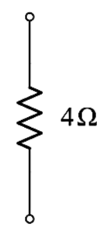
On the right branch, $4 + 1 = 5\,\Omega$



The two parallel resistors combine as $4.375\,||\,5 = \dfrac{4.375 \cdot 5}{4.375 + 5} = 2.33\,\Omega$

And we finish by adding the last two series resistors together,

$$R_{equivalent} = 2.33 + 1.66 = 4\,\Omega$$



Thevenin's Theorem

Any two-terminal linear network containing linear impedances and generators can be replaced with an equivalent circuit consisting of a voltage source E' in series with an impedance Z'. The value of E' is the open circuit voltage between the terminals of the network, and Z' is the impedance measured between the terminals with all other generators being removed, (but not their internal impedances).

Consider a two terminal network, containing one active mesh and other passive mesh. Let us also assume that load impedance Z_R is appearing between two terminals as shown in fig. (1a). Figure (1b) represents the Thevenin's equivalent network.

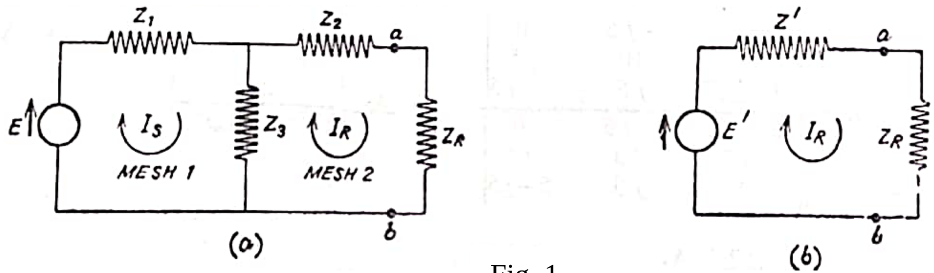


Fig. 1.

In fig. (1a) I_S and I_R represent the loop currents flowing in active and passive network respectively. Our aim is to calculate E' and Z' from fig. (1a) and then to show that it is equivalent to a circuit shown in fig. (1b). Applying the voltage law equations in mesh 1 and mesh 2 respectively, we have

$$Z_1 I_S + (I_S - I_R) Z_3 = E$$

$$Z_1 I_S + Z_3 I_S - Z_3 I_R = E \tag{1}$$

$$(Z_2 + Z_3 + Z_R) I_R - Z_3 I_S = 0 \tag{2}$$

From eq. (1),

$$I_S = \frac{E + Z_3 I_R}{Z_1 + Z_3}$$

Or Substituting the value of I_S in equation (2), we get

$$(Z_2 + Z_3 + Z_R) I_R - \frac{Z_3(E + Z_3 I_R)}{Z_1 + Z_3} = 0$$

$$(Z_2 + Z_3 + Z_R) I_R - \frac{Z_3^2 I_R}{Z_1 + Z_3} = \frac{Z_3 E}{Z_1 + Z_3}$$

$$\left(Z_2 + Z_3 + Z_R - \frac{Z_3^2}{Z_1 + Z_3}\right) I_R = \frac{Z_3 E}{Z_1 + Z_3}$$

$$I_R = \frac{\frac{Z_3 E}{Z_1 + Z_3}}{Z_2 + Z_R + Z_3 \frac{Z_3^2}{Z_1 + Z_3}}$$

$$I_R = \frac{\frac{Z_3 E}{Z_1 + Z_3}}{Z_2 + Z_R + \frac{Z_1 Z_3}{Z_1 + Z_3}}$$

The two equations (1) and (2) can also be easily solved with the help of Cramer's rule. The current flowing through load impedance Z_R is given by

$$(Z_1 + Z_3)I_S - Z_3 I_R = E \qquad (1')$$

$$-Z_3 I_S + (Z_2 + Z_3 + Z_R)I_R = 0 \qquad (2')$$

$$I_R = \frac{\begin{vmatrix} (Z_1 + Z_3) & E \\ -Z_3 & 0 \end{vmatrix}}{\begin{vmatrix} (Z_1 + Z_3) & -Z_3 \\ -Z_3 & (Z_2 + Z_3 + Z_R) \end{vmatrix}}$$

$$= \frac{Z_3 E}{(Z_1 + Z_3)(Z_2 + Z_3 + Z_R) - Z_3^2}$$

$$= \frac{\frac{Z_3 E}{Z_1 + Z_3}}{Z_2 + Z_3 + Z_R - \frac{Z_3^2}{Z_1 + Z_3}}$$

$$I_R = \frac{\frac{Z_3 E}{Z_1 + Z_3}}{Z_2 + Z_R + \frac{Z_1 Z_3}{Z_1 + Z_3}} \qquad (3)$$

This is the value of load current. By inspection of fig. (1a) the open circuit voltage at ab terminals is

$$E' = E\left(\frac{Z_3}{Z_1 + Z_3}\right) = \left(\frac{E}{Z_1 + Z_3}\right)Z_3 \qquad (4)$$

and the impedance of the network measured between ab terminals with all emfs generators short circuited is

$$Z' = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} \qquad (5)$$

From equations (3), (4) and (5), it can be easily seen that

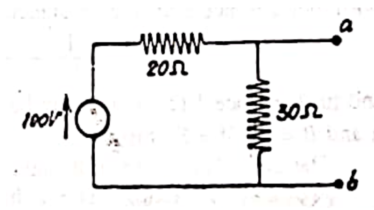
$$I_R = \frac{E'}{Z' + Z_R} \qquad (6)$$

Now from fig. (1b), we see that

$$I_R = \frac{E'}{Z' + Z_R} \qquad (7)$$

Equations (6) and (7) are the same, hence theorem has been proved for networks containing one generator. It may be generalized to any number of generators by the application of superposition theorem permitting each generator and associated circuit to be considered separately. E' and Z' as given by equations (4) and (5) are known as Thevenin's components.

Problem 1: Find the open circuit voltage and Thevenin resistance of the two-terminal network shown in fig.



Solution: We have E = 100 V, Z_1 = 20Ω. Z_2 = 0, Z_3 = 30 Ω. Then by Thevenin components we have open circuited voltage as

$$E' = E\left(\frac{Z_3}{Z_1 + Z_3}\right)$$

$$= 100\left(\frac{30}{20 + 30}\right) = 60\ V$$

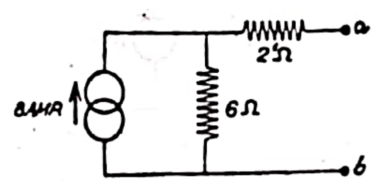
and

$$Z' = \frac{Z_1 Z_3}{Z_1 + Z_3}$$

$$= \frac{30 \times 20}{30 + 20} = 12\ \Omega$$

Hence $E' = 60\ V$ and Z' = 12 Ω.

Problem 2: Find the short circuit current and Thevenin resistance of the two-terminal network shown in fig.



Solution: As given in question, short circuit ab by connecting terminals ab with resistanceless wire, then applying current division law, we have

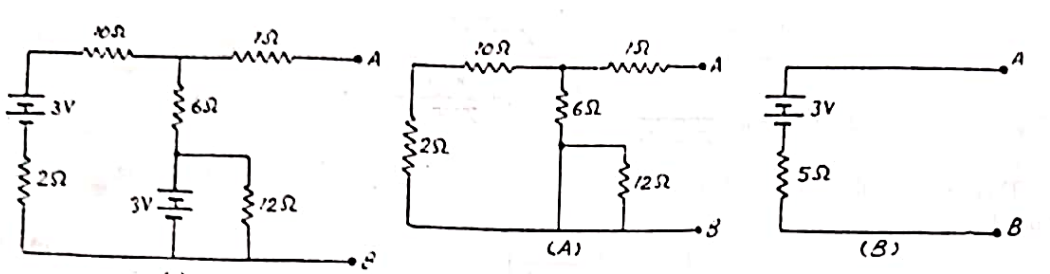
$$I_{ab} = \frac{6}{6 + 2} \times 8 = 6\ amp.$$

This is the short circuit current. Now for finding out Z', make current source as open circuit then

$$Z' = 2 + 6 = 8\,\Omega$$

Hence, short circuit current is 6 amp. and Z' = 8 Ω.

Problem 3: Obtain Thevenin's equivalent circuit for the network shown in fig.(a)



(a) (b)

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Solution: We shall first calculate the open-circuited voltage between A and B as follows: The loop eq. for big closed mesh may be written as

$$10\,I\ +6\,I+2\,I=3-3=0$$

$$I=0$$

Hence (i) current across 1Ω resistor will also be zero and (ii) voltage drop (= r x 1) across 6 **Ω** resistor and along 1**Ω** resistor will evidently be zero. Therefore, the voltage across A and B will be same as across 12 **Ω** resistor i.e., = 3 volt.

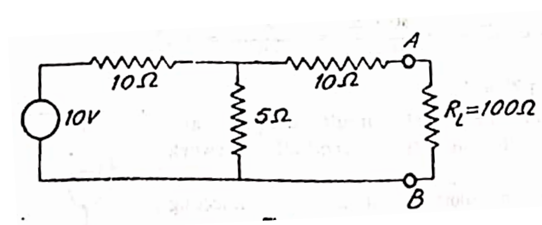
Now if each battery is replaced by zero resistance, as required for Thevenin's equivalent circuit. then given circuit reduces to that shown in fig. b(A). In this, it is evident that 12 **Ω** resistor becomes short circuited and so becomes ineffective. Therefore, the equivalent resistance across A and B may he computed as follows: 10 **Ω** and 2 **Ω** resistor may be supposed as connected in series and 6 **Ω** as in parallel with them. Hence effective resistance of these will be as given by

$$\frac{1}{R'}=\frac{1}{10+2}+\frac{1}{6}$$

$$R'=\frac{12}{3}=4\,\Omega$$

and further since 1 Ω resistor may be taken as connected in series with this, hence total resistance across A and B = 1 + 4 = 5 Ω. Therefore, Thevenin's equivalent circuit for given network may be shown as in fig. b (B) above.

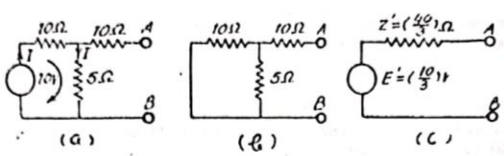
Problem 4: Using Thevenin's theorem, calculate the current in R_L in the given circuit (fig.).



Solution: First of all, we shall calculate the open circuited voltage E' across A and B in the absence of R_L. The loop equation is

$$10\,I+5I=10$$

$$I=\frac{10}{15}=\frac{2}{3}\,A$$



As no current flows through the resistor (10 **Ω**) near point A. hence voltage across it is zero. The voltage across AB is the same as voltage across 5 **Ω** resistor i.e.,

Voltage across AB = Voltage across 5 **Ω** resistor = current x resistance = $\frac{2}{3}\times 5=\frac{10}{3}$ V.

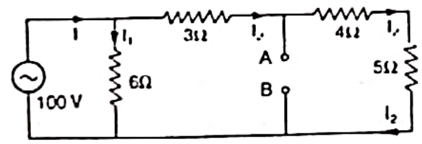
Now we shall calculate the impedance across AB, when the voltage source is replaced by zoo resistance as shown in fig. (b). As viewed across AB, 10 **Ω** and 5 **Ω** resistances are parallel then 10 **Ω** resistor is in series. Hence the impedance Z' between A and B is

$$Z' = 10 + \frac{10 \times 5}{10 + 5} = \frac{40}{3}\,\Omega$$

The Thevenin's circuit is shown in fig. (c). The current in the load, when connected across R_L is given by

$$I_L = \frac{\text{Voltage}}{\text{Resistance}} = \frac{\frac{10}{3}}{\frac{40}{3} + 100} = \frac{1}{34}A$$

Problem 5: Find the open-circuit voltage across AB of the two-terminal network own in fig. Also draw its Thevenin's equivalent circuit and calculate the current through a load impedance of 12.75 Ω when connected across A and B.



Solution:

In the given circuit, the resistances 3 Ω, 4 Ω and 5 Ω are in series while 6 Ω resistance is in parallel with them. Hence the resultant resistance is given by

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12}$$

$$R = 4\,\Omega$$

Now current $I = \frac{100}{4} = 25\,A$. The current I_2 through the series combination 3 Ω, 4 Ω and 5 Ω is

$$I_2 = \frac{6}{6+3+4+5} \times I = \frac{6}{18} \times 25 = \frac{25}{3}\ A$$

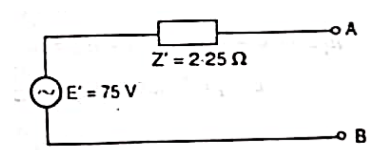
Potential difference across AB

$$E' = I_2(4+5) = \frac{25}{3} \times 9 = 75\,V$$

By short circuiting the source, the impedance across AB is given by

$$Z' = \frac{3(4+5)}{3+(4+5)} = 2.25\,\Omega$$

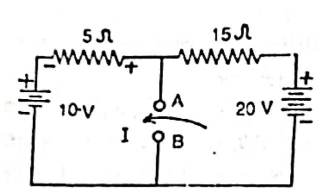
Here, the resistance 6 Ω has been short circuited. The Thevenin's equivalent circuit is:



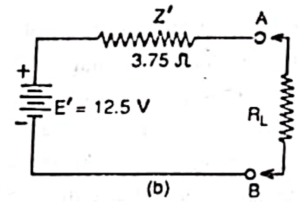
When a load of 12.75 is connected across AB, then the current through the load is given by

$$I_L = \frac{75}{2.25 + 12.75} = \frac{75}{15} = 5\,A$$

Problem 6: In the D.C. circuit shown in fig. three resistors, R_1 = 1 Ω, R_2 = 5 Ω and R_3 = 10 Ω, are connected in turns to terminals AB. Determine the power delivered to each resistor.



Solution: The Thevenin equivalent circuit is shown in fig.



Total current, $I = \frac{20-15}{5+15} = 0.5\ A$

Voltage-drop across 5 Ω resistor $= I \times 5 = 0.5 \times 5 = 2.5\ V$. Now

$$E_{AB} = E' = 10 + 2.5 = 12.5\ V$$

The impedance Z' across AB is a parallel combination of 5 Ω and 15 Ω. Therefore,

$$Z' = \frac{5 \times 15}{5 + 15} = \frac{75}{20} = 3.75\ \Omega$$

Now connecting each of the three resistors at terminal AB, the powers delivered are:

<u>with $R_L = 1\ \Omega$</u>, $I_1 = \frac{12.5}{3.75+1} = 2.63\ A$

$\therefore\ P_1 = I_1^2 \times R_L = 2.63^2 \times 1 = 6.91\ \boldsymbol{watt}.$

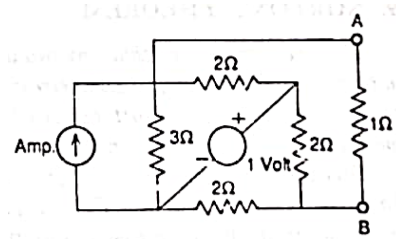
<u>with $R_L = 5\ \Omega$</u>, $I_2 = \frac{12.5}{3.75+5} = 1.43\ A$

$\therefore\ P_2 = I_2^2 \times R_L = 1.43^2 \times 5 = 10.2\ \boldsymbol{watt}.$

<u>with $R_L = 10\ \Omega$</u>, $I_3 = \frac{12.5}{3.75+10} = 0.91\ A$

$\therefore\ P_3 = I_3^2 \times R_L = 0.91^2 \times 10 = 8.28\ \boldsymbol{watt}.$

Problem 7: Determine the current in 1 Ω resistor across AB of the network shown in fig. using superposition and Thevenin's theorem.



Solution:

Here the open circuited voltage can be calculated with the help of superposition theorem. There are two sources and each is considered separately. Considering only 1 Amp. source. the equivalent circuit is shown in fig.1. Total resistance of loop PQRS

$$= \frac{3 \times 2}{3 + 2} = \frac{6}{5}\Omega$$

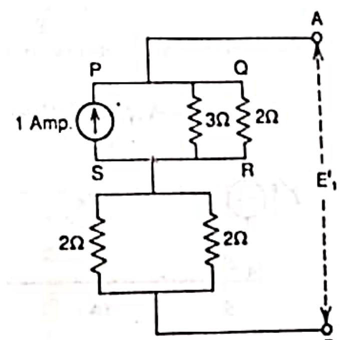


Fig.1

Now $E_1' = Current \times resistance = 1 \times \frac{6}{5} = 1.2\ V.$

No current flows through the lower loop containing two 2 Ω resistor. Further considering only 1 volt source, the equivalent circuit is shown in fig. (2 a and b). Applying Kirchhoff's voltage law to two loops of fig. 2(b), we get

$$i_1(2 + 2) = 1 \qquad or \quad i_1 = \frac{1}{4}$$

$$i_2(2 + 3) = 1 \qquad or \quad i_2 = \frac{1}{5}$$

$\therefore$ Voltage at X $= 3 \times \frac{1}{5} = 0.6\,V$

and voltage at Y $= 2 \times \frac{1}{4} = 0.5\,V$

Voltage across XY$= 0.6 - 0.5 = 0.1\,V$

Now $E'' = 1.2 + 0.1 = 1.3\,V$

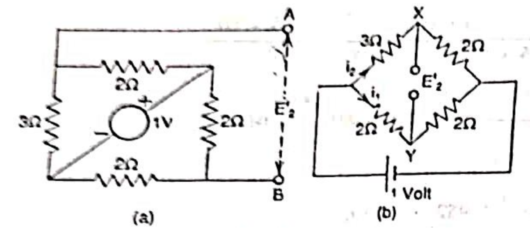


Fig.2

The value of Z" can be calculated with the help of fig. (3).

$$Z_{AB} = \frac{2 \times 3}{2+3} + \frac{2 \times 2}{2+2} = \frac{6}{5} + 1 = 2.2\,\Omega$$

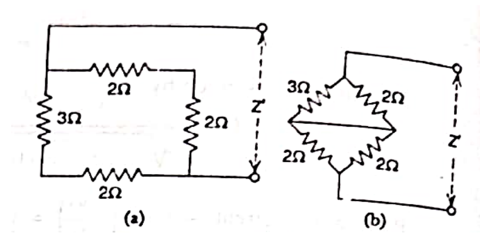


Fig.3

The Thevenin's equivalent circuit is shown in fig. (4). The current flowing through 1 Ω resistor is given by

$$I_{AB} = \frac{1.3}{2.2+1} = 0.406\,A = 406\,mA$$

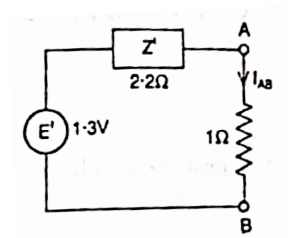


Fig.4

Admittance: admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the reciprocal of impedance, analogous to how conductance & resistance are defined. The SI unit of admittance is the siemens; the older, synonymous unit is mho, and its symbol is ℧.

Norton's Theorem

This theorem states that, any two-terminal network consisting of linear impedances and generators can be replaced by an equivalent circuit containing a current source I' in parallel with an admittance Y'. The value of I' is the short-circuited current between the terminals of the network, and Y' is the admittance measured between the terminals with all generators removed (but not their admittances).

This theorem may be easily proved by considering a Thevenin's equivalent network shown in fig. (a). The load impedance Z_R is appearing between two terminals a and b. Now this Thevenin's equivalent circuit may be easily converted into a circuit containing current source I' in parallel with Y' and Y_R appearing between two terminals a and b as shown in fig. (b).

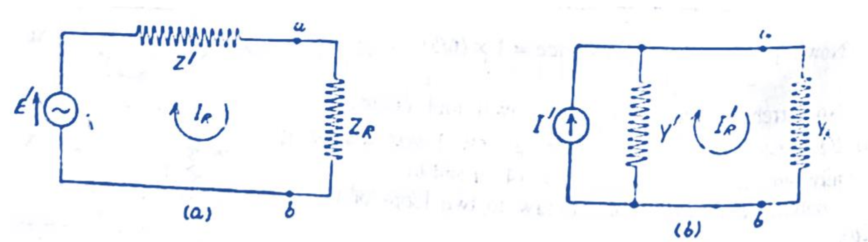


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Fig.1

The value of I_R from fig 1(a) is given by

$$I_R = \frac{E'}{Z' + Z_R} = \frac{E'}{\frac{1}{Y'} + \frac{1}{Y_R}} = \frac{E'Y'Y_R}{Y_R + Y'}$$

Why Y' and Y_R are the reciprocal of Z' and Z_R respectively, known as admittances.

Applying the current division law in fig. 1(b), we have

$$I_R{}' = \frac{I'Y_R}{Y' + Y_R{}'}$$

The load current $I_R{}'$ can be made equal to I_R. Then comparison of equations (1) and (2) gives

$$I' = E'Y' = \frac{E'}{Z'}$$

Equation (3) clearly indicates that circuits 1(a) and 1(b) are the same. Thus, we observe that fig.1(b) is equivalent to fig.1(b) and fig.1(a) is equivalent to the figure of Thevenin's equivalent circuit

Thus, we see that interchange of voltage and current sources with the help of Thevenin's and Norton's theorems gives a method of circuit analysis. As described earlier, voltage source is removed from a circuit by short circuiting its e.m.f. whereas a current source is removed by opening its circuit.

Problem 1: Convert the following linear network into Thevenin's equivalent network and then into Norton's equivalent network and show that power delivered to the load R_L in each case is same.

Solution: For converting this network into Thevenin's equivalent, we have to find out the Thevenin's components given as

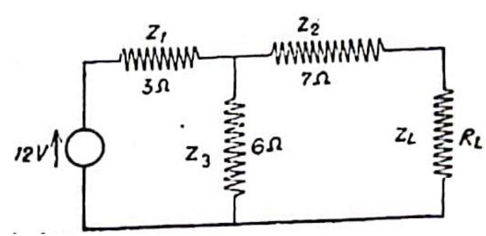


fig. a

$$E' = \frac{EZ_3}{Z_1 + Z_3}$$

$$= \frac{12 \times 6}{3 + 6} = 8\ volt$$

And

$$Z' = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}$$

$$= 7 + \frac{3 \times 6}{3 + 6} = 9\ \Omega$$

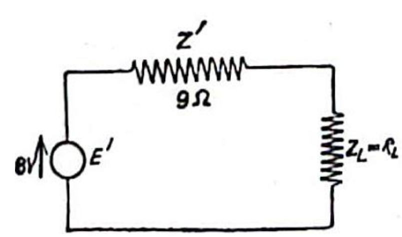


fig. b

Hence, the Thevenin's equivalent circuit is shown in fig. b and power delivered to the load is

$$P_{L_1} = (I_2)^2 R_L = \left(\frac{8}{9+R_L}\right)^2 R_L \quad (1)$$

Now applying Norton's theorem in fig. a, we have

$$I' = E'Y' = \frac{E'}{Z'} = \frac{8}{9} \ Amp$$

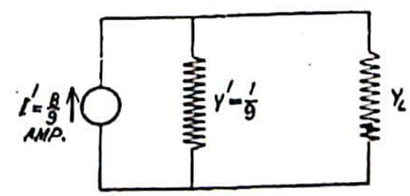


fig. c

And

$$Y' = \frac{1}{Z'} = \frac{1}{9} \ mho = \frac{1}{9} \ \mho$$

Hence the Norton's equivalent circuit is shown in fig. c. Now the power delivered to the load is calculated by applying current division law. The current through Y_L is given by

$$I_L = \frac{8}{9} \frac{Y_L}{\frac{1}{9} + Y_L}$$

So, the power,

$$P_{L_2} = (I_L)^2 R_L = (I_L)^2 \frac{1}{Y_L}$$

$$= \left(\frac{8}{9} \frac{Y_L}{\frac{1}{9} + Y_L}\right)^2 \frac{1}{Y_L}$$

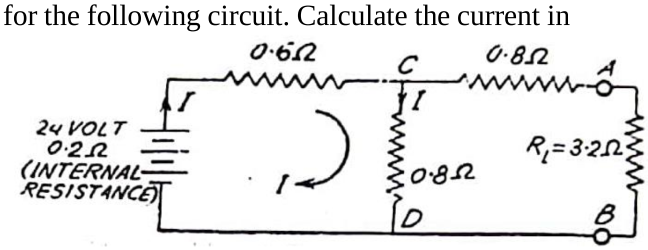
$$= \left(\frac{8}{9} \frac{\frac{1}{R_L}}{\frac{1}{9} + \frac{1}{R_L}}\right)^2 R_L$$

$$= \left(\frac{8}{9} \frac{\frac{1}{R_L}}{\frac{R_L + 9}{9R_L}}\right)^2 R_L = \left(\frac{8}{9} \frac{9R_L}{(R_L + 9)R_L}\right)^2 R_L$$

$$= \left(\frac{8}{9 + R_L}\right)^2 R_L \quad (2)$$

From eqn. 1 and 2 we see that in each cases power delivered to the load is the same and hence we conclude that both Thevenin's and Norton's circuits are equivalent to the original circuit.

Problem 2: Draw the Thevenin's and Norton's equivalent circuits for the following circuit. Calculate the current in the load in each case.



Solution: First of all, we shall calculate the open circuited voltage across AB. The current I flowing in the

loop is given by

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$$0.6I + O8I + 0.2I = 24 \quad i.e., \qquad I = 15\ A$$

The voltage. across AB is the same as voltage across CD. i.e.,

$$E' = voltage\ across\ CD$$

$$= 15\ A \times 0.8\ \Omega = 12\ Volt$$

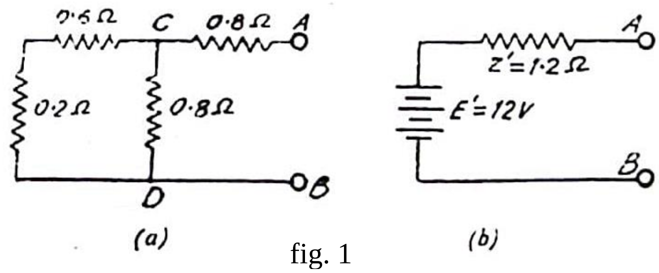


fig. 1

Now we shall calculate impedance across AB. For this purpose, the battery is removed but not its internal resistance. The circuit now becomes as shown in fig. (1a).

As viewed from points A and B, the resistors 0.2 Ω and 0.6 Ω are in series. Their equivalent resistance is 0.8 Ω. The resistor 0.8 Ω between C and D is in parallel. Hence equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{0.8} + \frac{1}{0.8} = \frac{1}{0.4}$$

$$\text{i.e. } R = 0.4\ \Omega$$

Now resistor 0.8 Ω between C and A is in series. Hence the impedance = 0.8 + 0.4 = 1.2 Ω. The Thevenin's equivalent circuit is shown in fig. (1b). The current in load R_L when connected between AB is given by

$$I_L = \frac{12V}{1.2\Omega + 3.2\Omega} = 2.73\ A$$

The Norton's equivalent circuit may be found from Thevenin's equivalent circuit. The short-circuited current found by shorting the terminals A and B together is

$$I_{AB} = \frac{12V}{1.2\Omega} = 10\ A$$

The Norton's equivalent circuit is shown in fig. (2).

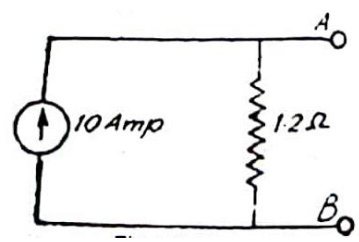
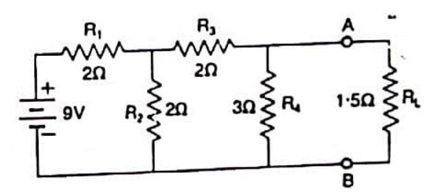


fig. 2

When the load R_L= 3.2 Ω is connected across AB, the current in the load is given by

$$I_L = \frac{1.2\ \Omega}{1.2\Omega + 3.2\Omega} \times 10\ A = 2.73\ A$$

Problem 3: Find the current flowing through resistor R_L in the network shown in fig. using Norton's theorem.



Solution: The impedance Z_{AB} between terminals AB can be obtained with the help of fig.

$$Z_{AB} = \frac{3 \times 3}{3 + 3} = 1.5\ \Omega$$

The current I drawn from voltage source can be found out by short circuiting AB. The given circuit now reduces as shown in fig. a

$$I = \frac{9}{2 + \frac{2 \times 2}{2 + 2}} = \frac{9}{3} = 3\ A$$

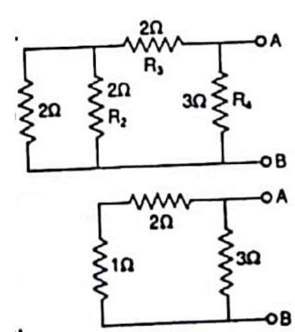


Fig. a

The current I' through short circuit is given by

$$I' = I\left(\frac{R_2}{R_2 + R_3}\right) = 3\left(\frac{2}{2+2}\right) = 1.5\ A$$

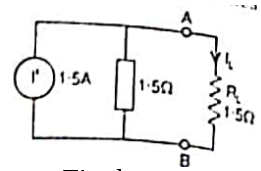


Fig. b

The Norton's equivalent circuit is shown in fig. b. The current I_L, flowing through R_L is given by

$$I_L = I'\left(\frac{1.5}{R_L + 1.5}\right) = 1.5\left(\frac{1.5}{1.5 + 1.5}\right) = 0.75\ A$$

Problem 4: Find the open circuit voltage and Thevenin's resistance of the two-terminal network shown in fig. and then reduce the network into Norton's equivalent circuit, and find the current through load resistance $Z_R = 2\ \Omega$. Find also the power delivered to the load.

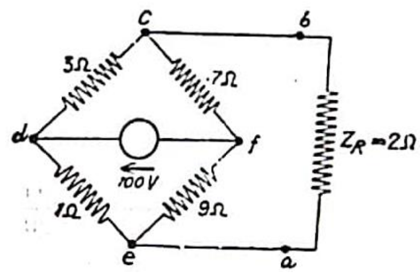


Fig. a

Solution: The Thevenin's resistance 7 can be calculated by short circuiting *df* branch. In that case *cd* and *cf* will be parallel to each other; and *de* and *fe* will be parallel to each other and both will he in series then

$$Z' = \frac{R_{cd} \times R_{cf}}{R_{cd} + R_{cf}} + \frac{R_{de} \times R_{ef}}{R_{de} + R_{ef}}$$

$$Z' = \frac{3 \times 7}{3+7} + \frac{1 \times 9}{1+9} = 3\ \Omega$$

Now we have to calculate the open circuited voltage between terminals *a* and *b*. We see from fig. a that the voltage drop between *ab* is same as at *c* and *e* points. The current generated from the source in each branch is voltage divided by the total resistance, i.e.

$$\frac{100}{10} = 10\ A$$

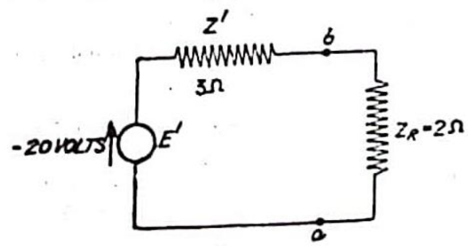


Fig. b

Potential difference across $cd = -30\ volt$, Potential difference across $d\ e = 100\ volt$, Potential difference across $ce = -20\ volt$. Calculating potential difference across *cf* and *fe* we would get potential difference across *ce* as $-20\ volt$.

$$E' = -20\ volt\ and\ Z' = 3\ \Omega$$

Now the circuit reduces into the Thevenin's equivalent as shown in fig. b. Now we have to convert this network into Norton's equivalent circuits. For this let us find out I', Y' and Y_R.

$$Y' = \frac{1}{Z'} = \frac{1}{3}\ \mho$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{2}\ \mho$$

$$I' = E'\,Y' = -20 \times \frac{1}{3} = -\frac{20}{3}$$

So, the Norton's circuit is as shown in fig. c. Current through the load is given by

$$I_R = \frac{I'Y_R}{Y' + Y_R} = -\frac{\frac{20}{3} \times \frac{1}{2}}{\frac{1}{3} + \frac{1}{2}} = -4\ A$$

This is in *ba* direction, hence current in *ab* direction is 4 *A*. Therefore, power delivered to the load is

$$P_L = \frac{I^2}{Y_R} = \frac{4^2}{\frac{1}{2}} = 32\ W$$

P = true power $\quad P = I^2R \quad P = \frac{E^2}{R}$

Measured in units of ***Watts***

Q = reactive power $\quad Q = I^2X \quad Q = \frac{E^2}{X}$

Measured in units of ***Volt-Amps-Reactive (VAR)***

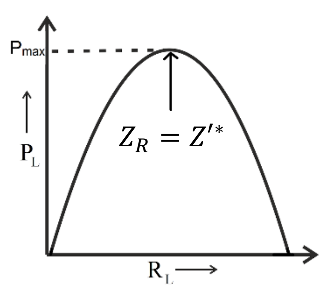
S = apparent power $\quad S = I^2Z \quad S = \frac{E^2}{Z} \quad S = IE$

Measured in units of ***Volt-Amps (VA)***

The Maximum Power Transfer Theorem

This theorem states that, the maximum power will be delivered by a network to a load impedance Z_R, if the impedance of Z_R is the complex conjugate of the impedance Z' of the network measured looking back into the terminals of the network. i.e.,

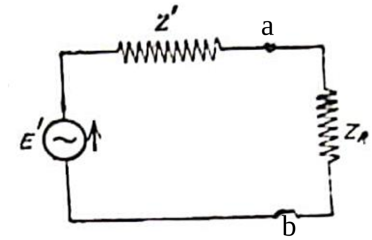
$$Z_R = Z'^*$$



P_{max} = Maximum Power consumption

The maximum power will be consumed by a network from another circuit connected to its two terminals when the impedance of the receiving network is varied to make the impedance looking into the network at its two terminals as conjugate to each other.

Let us consider a two terminal active linear network connected to a two terminal passive linear network. In fig. Z_R represents the equivalent impedance of a passive linear network and the network at the left of a, b terminals represent the Thevenin's equivalent active network.



The impedance of the active network Z' is equal to the ohmic resistance R' plus the resistance X'. i.e.,

$$Z' = R' + jX' \qquad (1)$$

Similarly

$$Z_R = R_R + jX_R \qquad (2)$$

We have to prove that Z' and Z_R arc conjugate to each other in order to transfer the maximum power to load. i.e.

$$Z_R = R' - jX' \qquad (3)$$

For this, we proceed as follows:

Let I be the current flowing in the network, then

$$I = \frac{E'}{Z' + Z_R}$$

$$I = \frac{E'}{(R' + jX') + (R_R + jX_R)}$$

$$= \frac{E'}{(R' + R_R) + j(X' + X_R)}$$

$$= \frac{E'}{\sqrt{(R' + R_R)^2 + (X' + X_R)^2}} \qquad (4)$$

The passer delivered to the load is

$$P_L = I^2 R_R = \frac{(E')^2}{(R' + R_R)^2 + (X' + X_R)^2} R_R \qquad (5)$$

If X_R is varying, the maximum power will be calculated by putting $\frac{\partial P}{\partial X_R} = 0$. From equation 4, we get

$$\frac{\partial P_L}{\partial X_R} = \frac{\partial}{\partial X_R}\left\{\frac{(E')^2}{(R' + R_R)^2 + (X' + X_R)^2} R_R\right\} = 0$$

$$\Rightarrow \frac{\partial}{\partial X_R}[(E')^2 R_R\{(R' + R_R)^2 + (X' + X_R)^2\}^{-1}] = 0$$

$$\Rightarrow -(E')^2 R_R\{(R' + R_R)^2 + (X' + X_R)^2\}^{-2}. 2(X' + X_R). 1 = 0$$

$$\Rightarrow \frac{-(E')^2 R_R.2(X'+X_R).1}{\{(R'+R_R)^2+(X'+X_R)^2\}^2}=0$$

$$\Rightarrow (X'+X_R)=0$$

$$\Rightarrow X'=-X_R \qquad (6)$$

Substituting equation (6) in equation (5), we get

$$(P_L)_{max}=\frac{(E')^2 R_R}{(R'+R_R)^2} \qquad \text{[True power]} \qquad (7)$$

Now suppose R_R is also varying, then maximum power will be calculated by putting from equation (7)

$$\frac{\partial P_L}{\partial R_R}=\frac{\partial}{\partial R_R}\left\{\frac{(E')^2 R_R}{(R'+R_R)^2}\right\}=0$$

$$\Rightarrow \frac{(R'+R_R)^2(E')^2.1-(E')^2 R_R.2(R'+R_R).1}{(R'+R_R)^4}=0$$

$$\Rightarrow (R'+R_R)^2(E')^2=(E')^2 R_R.2(R'+R_R)$$

$$\Rightarrow (R'+R_R)=2R_R$$

$$\Rightarrow R'=R_R \qquad (8)$$

Using equations (6) and (8), we can write

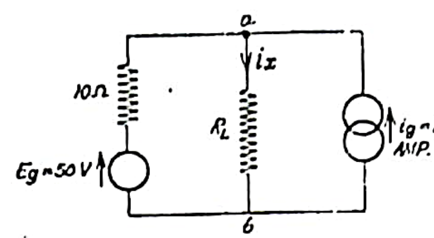
$$Z_R=R_R+jX_R=R'-jX'=Z'^*$$

Now from eqn. (7) and (8) the maximum power delivered to the load is

$$(P_L)_{max}=\frac{(E')^2 R_R}{(R_R+R_R)^2}=\frac{(E')^2}{4R_R}$$

Corollary: If only the absolute magnitude and not the angle of Z_R be varied, then the greatest power output be delivered from the network if the absolute magnitude of Z_R is made equal to the absolute magnitude of Z'.

Problem 1. Find the value of R_L which will absorb maximum power and determine this maximum power in the following network.



Solution: For finding out R_L, short circuit 50V source and open circuit the 1 amp. We see atonce that $R_L = 10\ \Omega$. Hence $R_L = 10\ \Omega$ will absorb maximum power from source.

For determining the maximum power absorbed by R_L we shall calculate the current flowing through R_L. We can find current through RL by superposition theorem.

(i) Let $i_g = 0$, then $i_{x_1} = \frac{50}{10+10} = 2.5\ amp$

(ii) Now let, $E_g = 0$, then $i_{x_2} = \frac{10}{10+10} \times 1 = 0.5\ amp$

Hence $i_x = i_{x_1} + i_{x_2} = 2.5 + 0.5 = 3\ amp$. Hence power absorbed by R_L is $P_{max} = {i_x}^2 R = 3^2 \times 10 = 90\ watt.$

References

- Hand Book of Electronics – Gupta & Kumar
- Basic Electronics – Bernard Grob
- khanacademy.org
- Internet