# Feedback and Oscillator Circuits

## Feedback

Feedback is a process in which a fraction of the output energy is combined to the input. Depending upon whether the feedback energy aids or opposes the input signal, there are two basic types of feedback in amplifiers i.e., **positive feedback** and **negative feedback**.

When the feedback voltage (or current) is so applied that it increases the input voltage (or current) i.e., it is in phase with the input, it is called **positive feedback** or **regenerative** or **direct feedback**. Positive feedback increases the gain of the amplifier. However, it has the disadvantage of increased distortion and instability. So positive feedback is seldom employed in amplifiers. If the positive feedback is sufficiently large, it leads to oscillations, and hence it is used in oscillators.

When the feedback voltage (or current's is so applied that it decreases the input voltage (or current) i.e., it is out of phase with the input. it is called **negative feedback** or **degenerative** or **inverse feedback**. Negative feedback reduces the gain of the amplifier. However, the advantages of negative feedback are: reduction in distortion, gain stability. increased bandwidth etc. So, negative feedback is frequently used in amplifier circuits.

## Principle of Feedback Amplifiers

For an ordinary amplifier, i.e., without feedback, let $\boldsymbol{V_o}$ and $\boldsymbol{V_i}$ be the output voltage and input voltage respectively. If $\boldsymbol{A}$ be the voltage gain of the amplifier, then

$$A = \frac{V_o}{V_i} \tag{1}$$

The gain A is often called open-loop gain.

The principle of an amplifier with feedback is shown in fig. The amplifier has two parts: an amplifier and a feedback circuit. Let $V_o'$ be the output voltage with feedback and a fraction $B$ of this voltage is applied to the input voltage. Now the input voltage becomes $(V_i \pm BV_o')$ depending on whether the feedback is positive or negative. This voltage is amplified $A$ times by the amplifier. Considering positive feedback, we have

$$A(V_i + BV_o') = V_o'$$

$$AV_i + ABV_o' = V_o'$$

$$AV_i = V_o'(1 - AB)$$

$$\frac{V_o'}{V_i} = \frac{A}{1 - AB} \tag{2}$$

The left-hand side of eq. (2) represents the amplifier gain $A'$ or $A_f$ with feedback. i.e.,

$$A_f = A' = \frac{A}{1-AB} \qquad \textit{for positive feedback} \tag{3}$$

And $$A_f = A' = \frac{A}{1-(-AB)} = \frac{A}{1+AB} \qquad \textit{for negative feedback} \tag{4}$$

Here the term $BA$ is called as ***feedback factor,*** $BV_o'$ is the ***feedback voltage*** and $B = \frac{V_f}{V_o'}$ as ***feedback ratio***. The term $(1 \pm BA)$ is known as ***loop gain*** and amplifier gain $A'$ with feedback is ***closed-loop gain*** (feedback loop is closed).

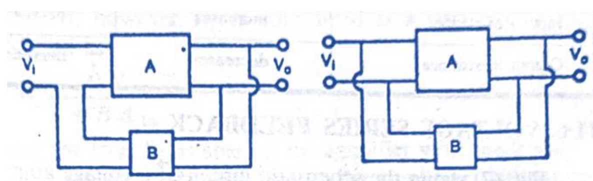
**Advantages of Negative Feedback** Following are the advantages of negative feedback:

- (i) Highly stabilized gain.
- (ii) Reduction in non-linear distortion.
- (iii) Increased bandwidth i.e., improved frequency response.
- (iv) Increased circuit stability.
- (v) Less amplitude distortion.
- (vi) Less frequency distortion.
- (vii) Less phase distortion.
- (viii) Less harmonic distortion.
- (ix) Reduce noise.
- (x) Increases input impedance and decreases output impedances i.e., input and output imped, modified as desired.

## Negative Feedback

Negative feedback in an amplifier is a method for feeding a portion of the amplified output energy hack to the input of the amplifier to oppose the input signal, There are two types of negative feedback circuits i.e., (a) negative voltage feedback and (b) negative current feedback.

**(a) Negative voltage feedback**. In this method, the voltage feedback to the input of the amplifier is proportional to the output voltage. This is further classified as: (i) voltage series feedback [fig. (a)] and (ii) voltage-shunt feedback [fig. (b)].

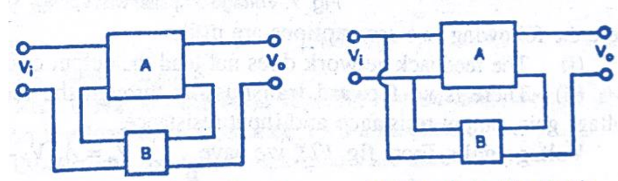


(a) Voltage series feedback. (b) Voltage-shunt feedback.

**i) Voltage-series feedback**. This is also known as shunt-derived series feedback. The amplifier circuit and feedback circuit are connected in series-parallel (sp). Here the output voltage is combined in series with the input voltage via feedback. As seen, the feedback network shunts the output but is in series with the input, hence output impedance decreases (parallel combination) while the input impedance increases (series combination) due to feedback.

**(ii) Voltage-shunt feedback**. This is also known as shunt-derived shunt-fed feedback i.e., a parallel-parallel (pp) prototype. Here a fraction of output voltage is combined with the input voltage in parallel (shunt). As seen, the feedback network shunts the output as well as input. hence both impedances (input and output) decrease due to feedback.

**(b) Negative current feedback**. In this method, the voltage feedback to the input of the amplifier is proportional to the output current. This is further classified as (i) current-series feedback [fig. (a.)] and (ii) current-shunt feedback [fig. (b)].



(a) Current series feedback. (b) Current-shunt feedback.

**(i) Current-series feedback**. This is also known as series derived series fed feedback i.e., a series-series (ss) circuit. Here a part of the output current feedbacks a proportional voltage in series with the input. As seen, the feedback network is in series with the input as well as output, and hence both the impedances (input and output) increase due to feedback.

**(ii) Current-shunt feedback.** This is also known as series derived shunt fed feedback i.e., a series-parallel (sp) circuit. Here a part of the output current is feedback a proportional voltage in parallel with the input voltage. As seen, the feedback network is in series with output and. in parallel with input, and hence the output impedance is increased while the input impedance is decreased.

## Reasons for Negative Feedback

### (a) Increased Stability

The gain of the amplifier with negative feedback is given by

$$A' = \frac{A}{1 + AB} \tag{1}$$

In negative feedback amplifiers, the designer deliberately makes the product $BA$ much greater than unity so that 1 may be neglected in comparison to it. Hence

$$A' \cong \frac{A}{AB} \tag{2}$$

Thus $A'$ depends only on $B$ (feedback ratio) i.e., characteristics of the feedback circuit. As a feedback circuit is usually a voltage divider (resistive network) and resistors can be selected very precisely with almost zero temperature coefficient of resistance, therefore the gain is unaffected by changes in temperature, variations in transistor parameters, and frequency. Hence the gain of the amplifier is extremely stable.

Let us consider the situation in which there is a change in the gain of the amplifier due to some reasons. Taking logs of both sides of equation (1), we get

$$\log A' = \log A - \log(1 + AB) \tag{3}$$

Differentiating both sides, we get

$$\frac{dA'}{A'} = \frac{dA}{A} - \frac{BdA}{1 + AB}$$

$$\frac{dA'}{A'} = \frac{dA}{A}\left(1 - \frac{BA}{1 + AB}\right)$$

$$\left|\frac{dA'}{A'}\right| = \left|\frac{dA}{A}\right| \frac{1}{|1 + AB|} \tag{4}$$

where $B\ A \gg\ 1$, we get

$$\left|\frac{dA'}{A'}\right| = \left|\frac{dA}{A}\right| \frac{1}{|AB|} \tag{5}$$

This expression shows that even in this case, there is an improvement in the stability of the gain. This will be clearer by considering the following example:

Let an amplifier has an open-loop gain of $400$ and feedback is $0.1$. If open-loop gain changes by $20\%$ due to temperature, find the percentage change in closed-loop gain.

Here

$$\left|\frac{dA'}{A'}\right| = 20\% \frac{1}{0.1 \times 400} = 0.5\%$$

So, when the amplifier gain changes by 20% the feedback gain changes by only 0.5% i.e., an improvement of $\frac{20}{0.5} = 40\ times.$

## (b) Reduction in Non-Linear Distortion

A large signal stage has non-linear distortion because its voltage gain changes at various points in the cycle. The use of negative feedback in large-signal amplifiers reduces the non-linear distortion.

Let $D$ = Distortion voltage generated in amplifiers without feedback
$D'$ = Distortion voltage generated in amplifier with feedback

Suppose

$$D' = xD \tag{1}$$

Now a fraction of output distorted voltage feedback to input $BD' = BxD$. This voltage is amplified by the amplifier. The amplified distorted voltage will be $BxDA$. This is in antiphase with original distortion voltage $D$. So, the new distorted voltage $D'$ which appears in the output is

$$D' = D - BxDA \tag{2}$$

From Eqs. (1) and (2) we get

$$xD = D - BxDA$$

$$x(1 + BA) = 1$$

$$x = \frac{1}{1 + BA}$$

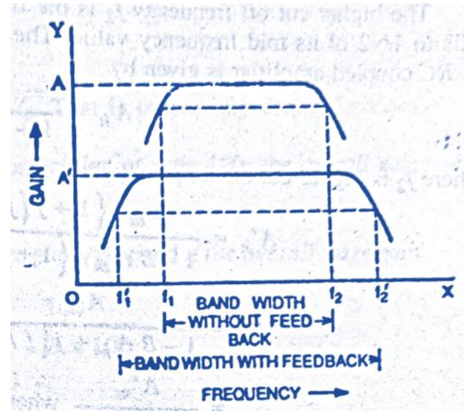
Substituting this value in Eq. (1), we get

$$D' = \frac{D}{1 + BA}$$

i.e., $D' < D$ So, the negative feedback reduces the amplifier distortion by a factor $(1 + BA)$. Here it should be remembered that the improvement in distortion is possible only when the distortion is produced by the amplifier itself and not when it is already present in the input signal.

## (c) Increased Bandwidth

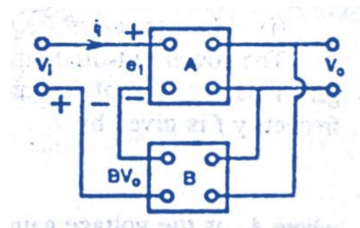
We have seen that amplifier gain falls off at low and series capacitances can no longer be taken as short-circuited and hence the gain falls off. At high frequencies, the shunt capacitances cannot be considered as open circuited as at mid frequencies, and hence due to the reactance of shunt capacitances, the amplifier gain falls off. Let $f_1$, and $f_2$ be the lower $3db$ frequency and upper $3db$ frequency respectively without feedback. Then the bandwidth of the amplifier will be $(f_2 - f_1)$ The bandwidth is shown in fig. If $A$ be the gain of the amplifier, then the gain-bandwidth product will be $A \times bandwidth$.




When feedback is applied, the gain of the amplifier is decreased but gain bandwidth remains the same. This indicates that the bandwidth must increase to compensate for the decrease in gain.

## (D) Effect on Input Impedance of a Transistor Amplifier

In order to consider the effect of feedback on input impedance of a transistor amplifier, we assume that $A$ is the normal gain of the amplifier without feedback. $BV_o$ is the fraction of the output voltage which is feedback to the input terminals as shown in fig. Without feedback, the input impedance is




$$Z_i = \frac{e_1}{i_1} = \frac{V_1}{i_1} \qquad \text{(since } V_1 = e_1\text{)}$$

With feedback, the input impedance $Z_{if}$ is given by ei

$$Z_{if} = \frac{e_1 - BV_o}{i_1}$$

$$= \frac{e_1 - BAe_1}{i_1} \qquad (V_o = Ae_1)$$

$$= \frac{e_1}{i_1}(1 - BA)$$

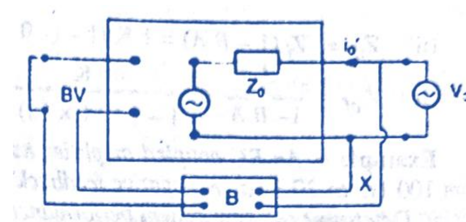
$$Z_{if} = Z_i(1 - BA)$$

In negative feedback, $(1 - BA)$ is greater than unity and consequently, $Z_{if}$ is greater than $Z_i$. That is due to negative feedback, input impedance a transistor amplifier increases.

## (e) Effect of Output Impedance of A Transistor Amplifier

The output impedance without feedback is given by

$$Z_o = \frac{V_o}{i_o}$$




In order to find out the output impedance of the amplifier with feedback, we short circuit the input source and connect a voltage source $V_o'$ at the output



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terminals as shown in fig. The output has been replaced by an equivalent voltage source $ABV_o$. Let $i_o'$ be the current with feedback. From figure,

$$Z_o i_o' = V_o - ABV_o$$

$$i_o' = \frac{V_o - ABV_o}{Z_o} = \frac{V_o(1 - AB)}{Z_o}$$

So the output impedance is

$$Z_{of} = \frac{V_o}{i_o'} = \frac{V_o}{\frac{V_o(1 - AB)}{Z_o}}$$

$$= \frac{Z_o}{1 - AB}$$

Since in negative feedback $(1 - AB) > 1$, $Z_{of}$ is less than $Z_o$. That is output impedance decreses due to negative feedback.

## Transistor Oscillator

An oscillator is a circuit which produces a continuous, repeated, alternating waveform without any input. Oscillators basically convert unidirectional current flow from a DC source into an alternating waveform which is of the desired frequency, as decided by its circuit components.

An amplifier produces an output signal whose waveform is similar to the input signal, of course, the power level is generally high. The amplifier is an energy converter i.e.,{ takes energy from a supply source and converts it into a.c. signal at the input signal frequency. When there is no input signal, there is no energy conversion i.e., there is no output signal. On the other hand, oscillator does not require an external input source and produces an output signal so long as d.c. power source is connected. Hence the oscillator may be defined as a circuit that generates an a.c. output signal without any externally applied input signal or a circuit that converts d.c. energy into a. c. energy at very high frequency. The output waveform may be sine, square, sawtooth, or pulse shapes.

Now the question is that an alternator (a.c. machine) can serve the purpose of an oscillator or not. The answer is no. The reason is that usually, an alternator generates frequencies up to $1000\ Hz$. To generate higher frequencies, there are so many practical difficulties. For example, either the number of poles has to be made large or the speed of rotation of armature has to be made extremely high. Both these factors are impracticable. Hence the alternator can not serve the purpose. So we have to depend on electronic circuits.

The network associated with the transistor determines the frequency of oscillations. While characteristics of the transistor with circuit determine to conditions of oscillations. The oscillators may be classified as:

(i) sinusoidal oscillators, and
(ii) relaxation oscillators.

Sinusoidal oscillators are those oscillators which operate on the linear portion of the characteristics, whereas the relaxation oscillators operate on the non-linear region of its characteristics. They are again classified as:

(i) feedback type, (ii) negative resistance type.

In feedback oscillators, part of the output is fed back to the input in proper phase and magnitude. In negative resistance oscillators, the transistor provides the negative resistance which cancels the positive resistance of the associated circuit and thus provides for oscillations.

The essential components of an oscillator are

**(i) Tank circuit**: The tank circuit consists of an inductance coil in parallel with a capacitor. The frequency of oscillations in the circuit depends upon the values of inductance and capacitance. This is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where L is the inductance of the induction coil. and C is the capacitance of the capacitor.

**(ii) Transistor amplifier:** It receives d.c. power from the battery and changes it into a.c. power for supplying it to the tank circuit. The oscillations of tank circuit are fed to the transistor amplifier which are amplified due to transistor amplifying action.

**(iii) Feedback Circuit:** The feedback circuit supplies a part of the output energy to the tank circuit in the correct phase to overcome the losses occurring in the tank circuit and the balance is radiated out in the form of electromagnetic waves. The feedback circuit provides positive feedback.

https://www.electronicshub.org/oscillator-basics/

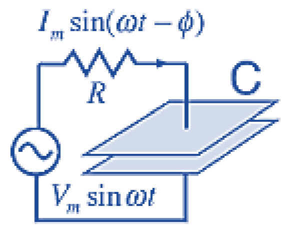
RC Oscillator Circuit - The RC Oscillator Tutorial (electronics-tutorials.ws)

$$h_{ie} = \left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}} = input\ impedance$$

$$h_{re} = \left(\frac{\partial V_{BE}}{\partial V_{CE}}\right)_{I_B} = reverse\ voltage\ ratio$$

$$h_{fe} = \left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}} = forward\ current\ transfer\ ratio$$

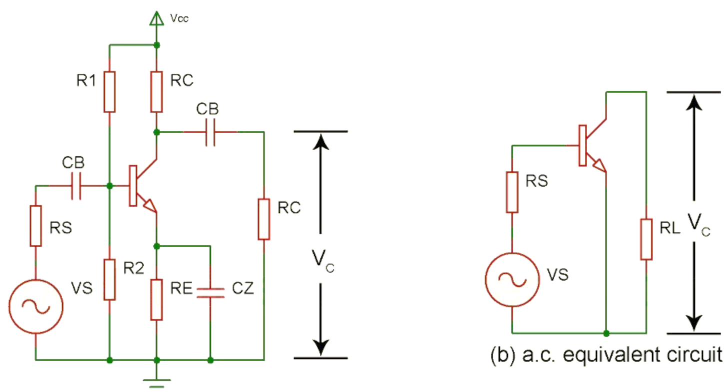
$$h_{oe} = \left(\frac{\partial I_C}{\partial V_{CE}}\right)_{I_B} = output\ admittance$$




$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

at phase:

$$\phi = \tan^{-1} \frac{-1/\omega C}{R}$$




(b) a.c. equivalent circuit

(a) Circuit of CE Amplifier

Figure 1: CE Amplifier

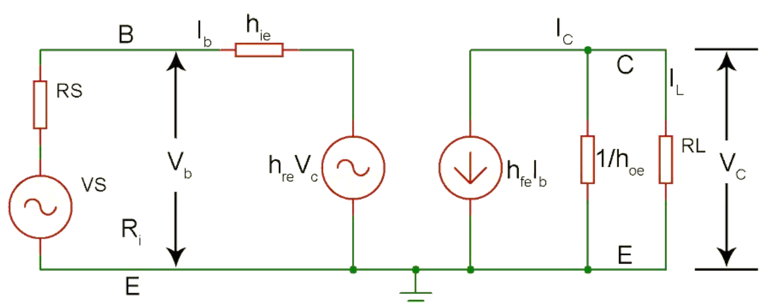
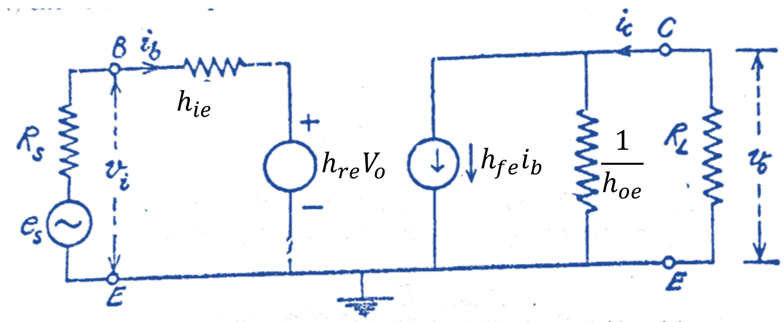



Figure 2: Equivalent Circuit of CE amplifier using h-model

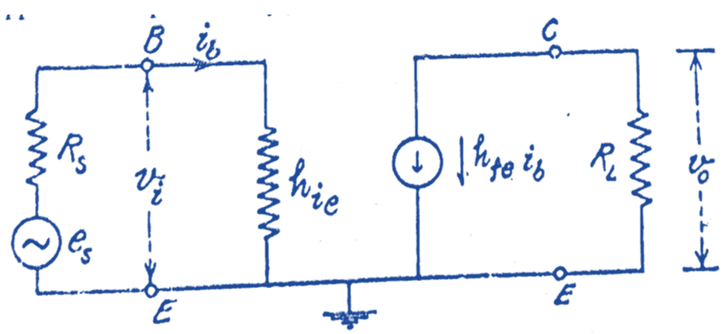
## Simplified Common Emitter Hybrid Model




Using the above equivalent circuit, we have calculated voltage gain, current gain, input impedance, and output impedance, etc. But in most practical cases, it is justified to obtain these values approximately rather than to calculate them exactly by lengthy and tedious analysis. This is true because h-parameters themselves vary widely for the transistor of the same type. The following approximations are taken

(i) As $\frac{1}{h_{oe}}$ is in parallel with $R_L$, the equivalent resistance is a parallel combination of $R_L$ and $\frac{1}{h_{oe}}$ which is approximately $R_L$ if $\frac{1}{h_{oe}} \gg R_L$ i.e., $h_{oe}R_L \ll 1$. So if this condition is satisfied. we may omit $\frac{1}{h_{oe}}$ from the circuit. Under this condition, the collector current $i_c = h_{fe}i_b$.

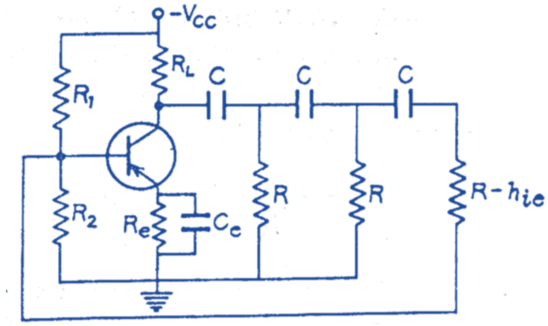
(ii) The magnitude of the voltage of the generator in the emitter circuit is $h_{re}v_o = h_{re}i_cR_L = h_{re}h_{fe}i_bR_L$. Since $h_{re}h_{fe} \approx 0.01$ and hence this voltage may be neglected in comparison with $h_{ie}i_b$, drop across $h_{ie}$, provided that $R_L$ is not too large. Thus, we conclude that if $R_L$ is small, then we can neglect $h_{re}$ and $h_{oe}$ from the exact equivalent circuit. Now the approximate equivalent hybrid model at low frequencies is




The simplified common-emitter hybrid model

## Phase Shift Oscillators

The phase shift transistor oscillator is similar to the vacuum tube phase shift oscillator. To obtain a positive feedback essential for oscillations, the frequency determining circuit must introduce a phase change of 180°. This phase shift of 180° is obtained with three cascade sections $CR, CR, CR$ (each section consists of a series coupling capacitor and a shunt resistor) each shifting the signal by 60°. The phase shift comes about because R and C provide a current which leads the applied voltage by a certain angle. The smaller is the capacitance more will the current lead the voltage for a given resistance. With a proper choice of R and C, a phase shift of 60° per section is achieved.




**Analysis**: For convenience, the following assumptions are made:

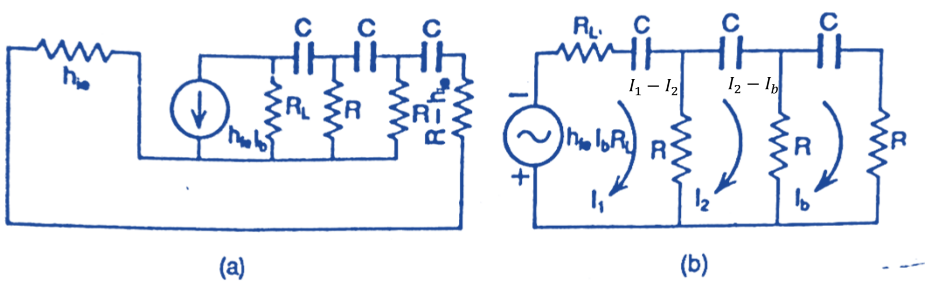
(i) The three $RC$ sections are made identical, of course, the third resistor is taken as $(R - h_{ie})$ because the input resistance of the transistor $h_{ie}$ is added to give a total resistance of R.

(ii) The biasing resistances $R_1$ and $R_2$ have no effect on the a.c, operation of the circuit since they are larger.

(iii) Since $\frac{1}{h_{oe}}$ is much larger than $R_L$, its effect can be neglected.

(iv) Since $h_{re}$ of the transistor is usually very small, $h_{re}v_o$can be neglected.

With all these considerations, the equivalent circuit is shown in fig below:




Applying Kirchhoff's voltage law for three loops in fig.

$$\left.\begin{aligned} I_1 R_L + I_1 \frac{1}{j\omega C} + (I_1 - I_2)R &= -h_{fe} I_b R_L \\ I_2 \frac{1}{j\omega C} + (I_2 - I_b)R - (I_1 - I_2)R &= 0 \\ I_b \frac{1}{j\omega C} + I_b R - (I_2 - I_b)R &= 0 \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} I_1\left(R + R_L + \frac{1}{j\omega C}\right) - I_2 R + I_b h_{fe} R_L = 0 \\ -I_1 R + I_2\left(2R + \frac{1}{j\omega C}\right) - I_b R = 0 \\ -I_2 R + I_b\left(2R + \frac{1}{j\omega C}\right) = 0 \end{aligned}\right\} \tag{2}$$

The determinant form of the above equations is given by

$$\begin{vmatrix} R + R_L + \frac{1}{j\omega C} & -R & h_{fe} R_L \\ -R & 2R + \frac{1}{j\omega C} & -R \\ 0 & -R & 2R + \frac{1}{j\omega C} \end{vmatrix} = 0$$

$$\left(R + R_L + \frac{1}{j\omega C}\right)\left\{\left(2R + \frac{1}{j\omega C}\right)^2 - R^2\right\} - R\left(0 + 2R^2 + \frac{R}{j\omega C}\right) + h_{fe} R_L (R^2 - 0) = 0$$

$$\left(R + R_L + \frac{1}{j\omega C}\right)\left\{4R^2 + \frac{4R}{j\omega C} - \frac{1}{\omega^2 C^2} - R^2\right\} - R\left(0 + 2R^2 + \frac{R}{j\omega C}\right) + h_{fe} R_L (R^2 - 0) = 0$$

$$\left\{(R + R_L) + \frac{1}{j\omega C}\right\}\left\{\left(3R^2 - \frac{1}{\omega^2 C^2}\right) + \frac{4R}{j\omega C}\right\} - 2R^3 - \frac{R^2}{j\omega C} + h_{fe} R_L R^2 = 0$$

$$(R + R_L)\left(3R^2 - \frac{1}{\omega^2 C^2}\right) + (R + R_L)\frac{4R}{j\omega C} + \frac{1}{j\omega C}\left(3R^2 - \frac{1}{\omega^2 C^2}\right) - \frac{4R}{\omega^2 C^2} - 2R^3 - \frac{R^2}{j\omega C} + h_{fe} R_L R^2 = 0 \tag{3}$$

Equating the imaginary part to zero, we get

$$(R + R_L)\frac{4R}{\omega C} + \frac{1}{\omega C}\left(3R^2 - \frac{1}{\omega^2 C^2}\right) - \frac{R^2}{\omega C} = 0$$

$$(R + R_L)4R + 3R^2 - \frac{1}{\omega^2 C^2} - R^2 = 0$$

$$4R^2 + 4RR_L + 3R^2 - R^2 - \frac{1}{\omega^2 C^2} = 0$$

$$\frac{1}{\omega^2 C^2} = 6R^2 + 4RR_L \tag{4}$$

$$\omega = \frac{1}{C\sqrt{6R^2 + 4RR_L}}$$

$$f = \frac{1}{2\pi C\sqrt{6R^2 + 4RR_L}} \tag{5}$$

This equation gives the frequency of oscillations. To obtain the condition of maintenance of oscillations, we compare the real part of Eq. (3) to zero.

$$(R+R_L)\left(3R^2-\frac{1}{\omega^2C^2}\right)-\frac{4R}{\omega^2C^2}-2R^3+h_{fe}R_LR^2=0$$

$$3R^3-\frac{R}{\omega^2C^2}+3R^2R_L-\frac{R_L}{\omega^2C^2}-\frac{4R}{\omega^2C^2}-2R^3+h_{fe}R^2R_L=0$$

Using Eq. 4 we get,

$$3R^3-R(6R^2+4RR_L)+3R_LR^2-R_L(6R^2+4RR_L)-4R(6R^2+4RR_L)-2R^3+h_{fe}R^2R_L=0$$

$$3R^3-6R^3-4R^2R_L+3R^2R_L-6R^2R_L-4RR_L{}^2-24R^3-16R^2R_L-2R^3+h_{fe}R^2R_L=0$$

$$-29R^3-23R^2R_L-4RR_L{}^2+h_{fe}R^2R_L=0$$

$$h_{fe}=29\frac{R^3}{R^2R_L}+23\frac{R^2R_L}{R^2R_L}+4\frac{RR_L{}^2}{R^2R_L}$$

$$h_{fe}=29\frac{R}{R_L}+23+4\frac{R_L}{R} \qquad (6)$$

Eq. (6) gives the condition for sustained oscillations. In practice, $R_L$ is taken equal to $R$. Hence from Eqs. (5) and (6), we have

$$f=\frac{1}{2\sqrt{10}\pi RC}$$

$$h_{fe}=29+23+4=56$$

So, for the phase shift oscillators with $R_L=R$, $h_{fe}$ should be 56 for sustained oscillations.

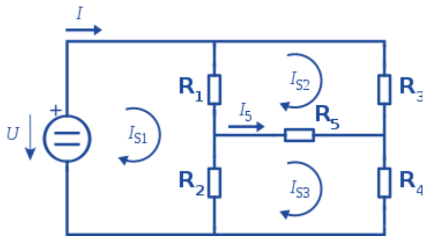
## Advantages

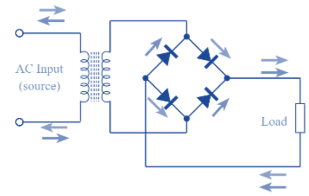
- It does not require transformers or inductors.
- It can be used to produce very low frequencies.
- The circuit provides good frequency stability.

## Disadvantages

- It is difficult for the circuit to start oscillations as the feedback is generally small.
- The circuit gives small output.

A bridge circuit is a topology of electrical circuitry in which two circuit branches (usually in parallel with each other) are "bridged" by a third branch connected between the first two branches at some intermediate point along them.

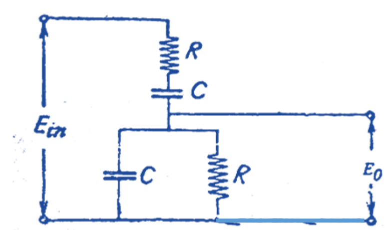



## Wien Bridge Oscillator

One of the simplest sine wave oscillators which use a $RC$ network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform, is called a Wien Bridge Oscillator. The Wien Bridge Oscillator is so-called because the circuit is based on a frequency-selective form of the Wheatstone bridge circuit. The Wien Bridge oscillator is a two-stage RC coupled amplifier circuit that has good stability at its resonant frequency, low distortion, and is very easy to tune making it a popular circuit as an audio frequency oscillator but the phase shift of the output signal is considerably different from the previous phase shift RC Oscillator.

This is also an audio frequency oscillator. The advantage of this oscillator is that the frequency may be varied over a frequency range of $1:10$, whereas in RC oscillators the frequency cannot be varied. i.e. $RC$ oscillator is a fixed-frequency oscillator. In. $RC$ oscillator both frequency-determining network and amplifier introduce a phase change in the signal and positive feedback is obtained. On the other hand, the oscillations may be obtained by arranging both network and amplifier to introduce zero phase shift, and actually, this is the principle of the Wien-bridge oscillator.




In the Wien-bridge network, the upper arm consists of resistance and capacitance in series, while the lower arm has some resistance and capacitance in parallel. The network is supplied from a constant voltage source and is terminated in an infinite impedance. The phase shift and attenuation introduced by the network can be calculated as follows:

The impedance of the parallel $RC$ network is

$$\frac{-R.\frac{j}{\omega C}}{R-\frac{j}{\omega C}}$$

and of the series RC network is

$$R-\frac{j}{\omega C}$$

$$\frac{E_0}{E_{in}}=\frac{impedance\ of\ parallel\ combination \times current}{total\ impedance\ \times current} \tag{1}$$

$$\frac{E_0}{E_{in}} = \frac{\dfrac{-R.\dfrac{j}{\omega C}}{R - \dfrac{j}{\omega C}}}{R - \dfrac{j}{\omega C} - \dfrac{R.\dfrac{j}{\omega C}}{R - \dfrac{j}{\omega C}}}$$

$$\frac{E_0}{E_{in}} = \frac{-R.\dfrac{j}{\omega C}}{\left(R - \dfrac{j}{\omega C}\right)^2 - \dfrac{Rj}{\omega C}} = \frac{-R.\dfrac{j}{\omega C}}{R^2 - 2\dfrac{Rj}{\omega C} - \dfrac{1}{\omega^2 C^2} - \dfrac{Rj}{\omega C}}$$

$$\frac{E_0}{E_{in}} = \frac{-R.\dfrac{j}{\omega C}}{R^2 - 3\dfrac{Rj}{\omega C} - \dfrac{1}{\omega^2 C^2}} \qquad (2)$$

The phase shift occurs when there is an imaginary part associated with the real part. So there is zero phase shift if the terms in $j$ vanishes, and that will occur in the above equation if

$$R^2 - \frac{1}{\omega^2 C^2} = 0$$

$$\omega = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC} \qquad (3)$$

Again from equation 2, using this condition we get,

$$\frac{E_0}{E_{in}} = \frac{-R.\dfrac{j}{\omega C}}{-3\dfrac{Rj}{\omega C}}$$

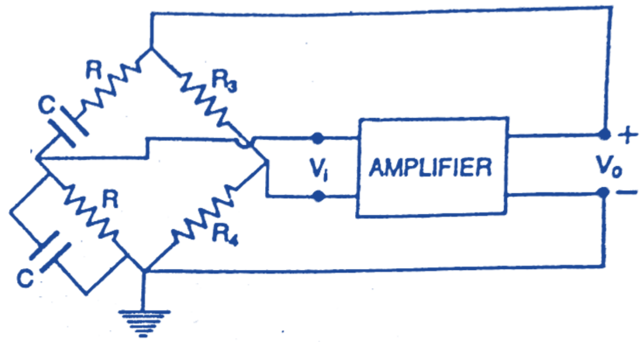
$$\frac{E_0}{E_{in}} = \frac{1}{3} \qquad (4)$$

As the output of this network is the input of the amplifier and vice versa, the maintaining amplifier thus requires a gain just exceeding 3 to sustain oscillations. As pointed out earlier, the output voltage $V_i$ from the bridge must not be zero for oscillation to occur. This can be achieved by taking the ratio $\frac{R_4}{R_3+R_4}$ smaller than (1/3). Thus,

$$\frac{R_4}{R_3 + R_4} = \frac{1}{3} - \frac{1}{K}$$

Where K > 3. The bridge now will be slightly unbalanced and gives a feedback voltage $V_i$. The Wien-bridge oscillator at balance

$$\frac{R_3}{R_4} = \frac{R - \frac{j}{\omega C}}{\frac{-\frac{Rj}{\omega C}}{R - \frac{j}{\omega C}}}$$




$$\frac{R_3}{R_4} = \frac{\left(R - \frac{j}{\omega C}\right)^2 \omega C}{-Rj}$$

$$\frac{R_3}{R_4} = \frac{\left(R - \frac{j}{\omega C}\right)^2 j\omega C}{R} = \frac{\left(R^2 - 2\frac{Rj}{\omega C} - \frac{1}{\omega^2 C^2}\right) j\omega C}{R}$$

$$\frac{R_3}{R_4} = \frac{j\omega C R^2 + 2R - \frac{j}{\omega C}}{R} = j\omega CR + 2 - \frac{j}{\omega CR}$$

Equating the imaginary term to zero, we get

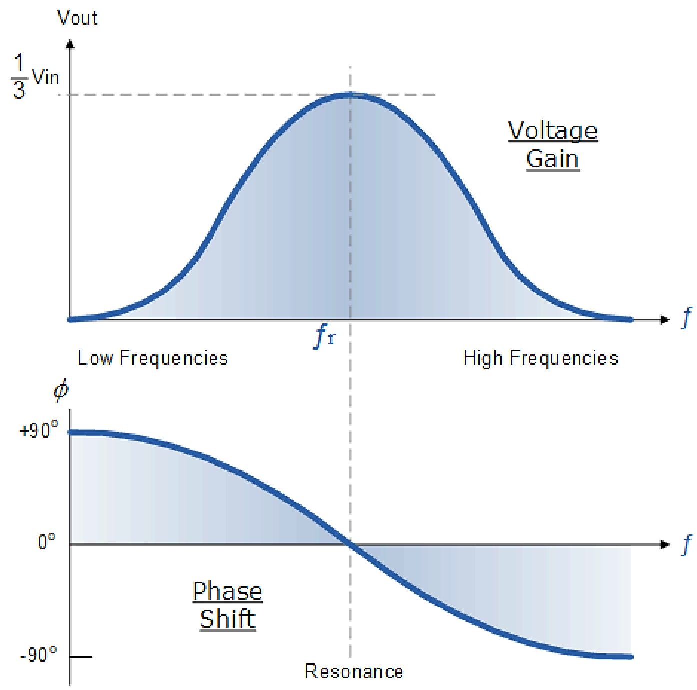
$$0 = \omega CR - \frac{1}{\omega CR}$$

$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\omega = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC}$$

Where $f$ is the frequency of oscillations.




**Problem 1**. A Wien bridge oscillator is to cover a frequency range from 20 Hz to 20 k Hz. The variable capacitance has a value from 30 pF to 300 pF. Calculate the resistance values required to cover the frequency range. If the gain of the amplifier is 5, find the ratio of the resistances in the other arms of the bridge.

**Solution:**

In the case of the Wien bridge oscillator, the frequency of oscillations is given by

$$f = \frac{1}{2\pi RC}$$

$$R = \frac{1}{2\pi f C}$$

We know that the capacitive reactance is maximum at a lower frequency, hence we can take $f = 20\ Hz$ and $C = 300\ pF$

$$R = \frac{1}{2 \times 3.1416 \times 20 \times 300 \times 10^{-12}}$$

$$= 26.5 \times 10^6\ \Omega = 26.5\ \mathrm{M\Omega}$$

It is obvious from the question that capacitance changes in the ratio $1:10$, Hence with the resistance $26.5\ \mathrm{M\Omega}$, a frequency range from $20\ Hz$ to $200\ Hz$ can be covered. For the next frequency range 200 Hz to 2 kHz, the resistance should be decreased by a factor of 10. Thus $R = 2.65\ \mathrm{M\Omega}$. Similarly, for the frequency range from $2\ kHz$ to $20\ kHz$ another resistance of $R = 0.265\ \mathrm{M\Omega}$ is required. Therefore, the three values of resistances are $0.265\ \mathrm{M\Omega}$, $2.65\ \mathrm{M\Omega}$, and $26.5\ \mathrm{M\Omega}$.

Given that the gain of the amplifier is 5. So, $K = 5$.

Now

$$\frac{R_4}{R_3 + R_4} = \frac{1}{3} - \frac{1}{K}$$

$$\frac{R_4}{R_3 + R_4} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$\frac{R_3 + R_4}{R_4} = \frac{15}{2}$$

$$1 + \frac{R_3}{R_4} = \frac{15}{2}$$

$$\frac{R_3}{R_4} = \frac{13}{2}$$

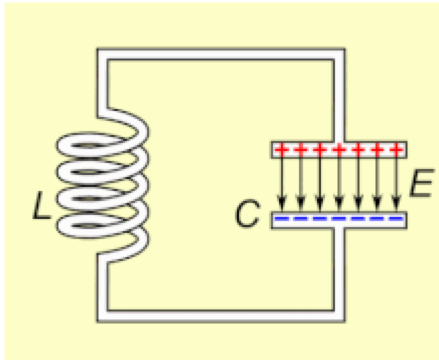
$$R_3 : R_4 = 13 : 2$$

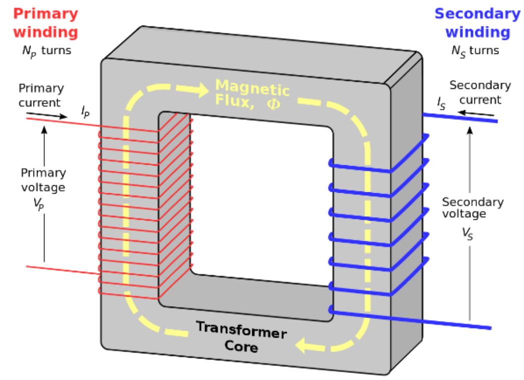
## Advantages of Wien Bridge Oscillator

- It gives constant output.
- The circuit works quite easily.
- The overall gain is high because of two transistors.
- The frequency of oscillations can be easily changed by using potentiometer.

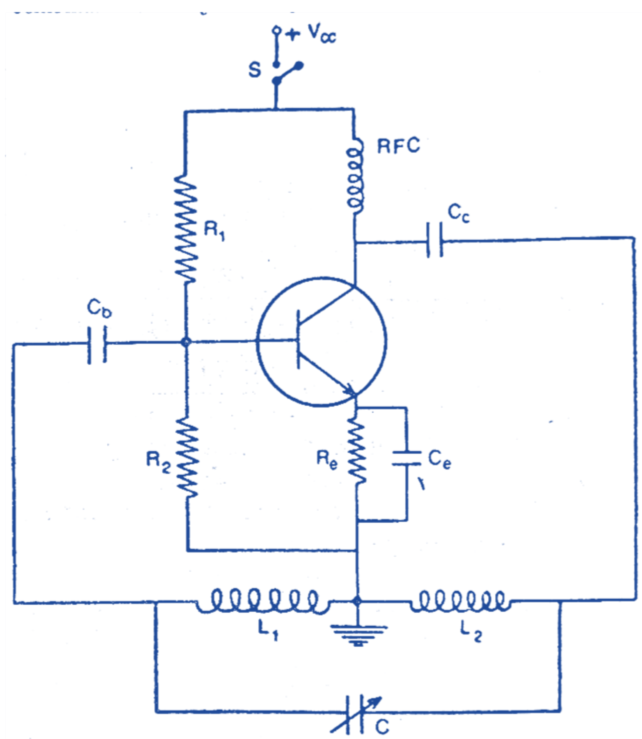
## Disadvantages of Wien Bridge Oscillator

- The circuit requires two transistors and a large number of components.
- It cannot generate very high frequencies




## Hartley Oscillator

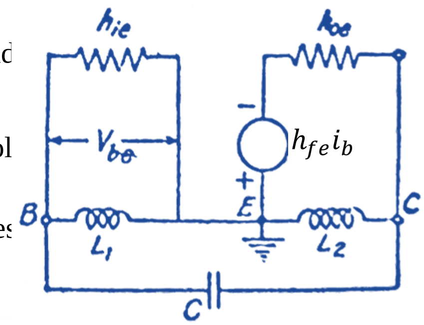



The figure shows a circuit of a shunt-fed Hartley oscillator using a transistor in $CE$ configuration. In this circuit, the parallel combination, of $R_e$ and $C_e$ in conjunction with $R_1$ and $R_2$ combination provides stabilized self-bias. The frequency determining network is made up of the variable capacitor C and the inductors $L_1$ and $L_2$. The coil $L_1$ is inductively coupled to the coil $L_2$ and the combination forms an auto-transformer. So far as a.c. signals are concerned, one side of $L_1$ is connected to the base via $C_b$ and the other to emitter via ground and $R_e$. Similarly, one end of $L_2$ is connected to the collector via $C_c$ and the other to common emitter via $C_e$. In this way $L_1$ is connected in the base-emitter circuit (input circuit) and $L_2$ is connected in the collector-emitter circuit (output circuit). Feedback between output and input is accomplished through transformer action. The transformer introduces a phase change of 180°. The transistor also introduces a phase change of 180°. Thus total phase change becomes 360°. This makes the feedback positive, which is the essential requirement for oscillations. Radio-frequency-choke (RFC) provides d.c. load for the collector and also keeps a.c. current out of the d.c. supply $V_{cc}$. The reactance of RFC is higher than $L_2$ and hence may be omitted from the equivalent circuit. The condenser $C_c$, blocks d.c. and provides an a.c. path from the collector to the tank circuit. It acts as an open circuit at zero frequency. The capacitor $C_b$ has a low reactance at the frequency of oscillations and may be omitted from the equivalent circuit.

### Circuit operation

When the switch S is closed, the collector current starts rising and charges the capacitor. When capacitor $C$ is fully charged, it discharges through coils $L_1$ and $L_2$. Now damped harmonic oscillations are set up in the tank circuit. The oscillations across $L_1$ are applied to the input circuit (base-emitter junction) and of Hartley oscillator. appear in the amplified form in the output circuit (collector circuit). Feedback of energy from the collector-emitter circuit to the base-emitter circuit is accomplished by means of mutual inductance between $L_1$ and $L_2$. In this way, energy is continuously supplied to the tank circuit to overcome the losses occurring in it. So continuous undamped output is obtained.

## General theory of Hartley Oscillator



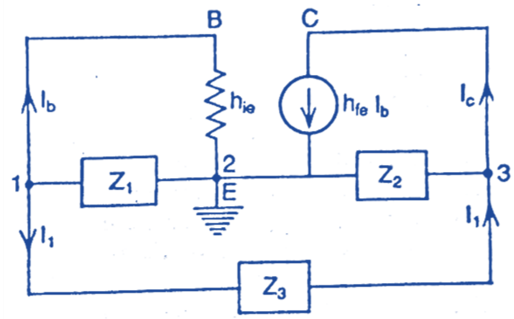

A complete theory of Hartley oscillator using a bipolar transistor is complicated anc assumptions :

a) The feedback source of e.m.f., $h_{re}V_o$, is omitted because $h_{re}$ (reverse vol negligible.

b) The output admittance $h_{oe}$ of the transistor is very small i.e., the output res parallel with the inductance $L_2$ is very large. Hence $\frac{1}{h_{oe}}$ is omitted.

c) The inductive and capacitive reactances are represented by $Z_1$, $Z_2$ and $Z_3$.

d) The input terminals are taken as 1 and 2 while the output terminals are taken as 2 and 3.




The load impedance $Z_1$ and input impedance $h_{ie}$ are in parallel and hence the equivalent impedance $Z_1{}'$ is given by

$$Z_1' = \frac{Z_1\, h_{ie}}{Z_1 + h_{ie}} \tag{1}$$

Now the load impedance $Z_L$ between output terminals 2 and 3 is equivalent to the equivalent impedance of $Z_2$ in parallel with the series combination of $Z_1{}'$ and $Z_3$. Hence

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{Z_1' + Z_3}$$

$$Z_L = \frac{Z_2\,(Z_1' + Z_3)}{Z_2 + Z_1' + Z_3} \tag{2}$$

The voltage gain without feedback is given by

$$A_{ve} = -\frac{h_{fe}\, Z_L}{h_{ie}} = \frac{\left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}} Z_L}{\left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}}} = \frac{\partial I_C\, Z_L}{\partial V_{BE}} = \frac{output\ voltage}{input\ voltage} \tag{3}$$

In order to obtain the feedback fraction $\beta$, we consider the output voltage between terminals 2 and 3. The output voltage is given by

$$V_0 = I_1(Z_1' + Z_3)$$

The voltage feedback to the input terminals 1 and 2 is given by

$$V_{fb} = I_1 Z_1'$$

$$\therefore \quad \beta = \frac{V_{fb}}{V_0} = \frac{Z_1'}{Z_1' + Z_3} \tag{4}$$

Applying the condition $A_{ve}\,\beta = 1$ for oscillation, we get

$$-\frac{h_{fe}\, Z_L}{h_{ie}} \times \frac{Z_1'}{Z_1' + Z_3} = 1$$

Substituting the value of $Z_L$ , we get

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\,(Z_1' + Z_3)}{Z_2 + Z_1' + Z_3} \times \frac{Z_1'}{Z_1' + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\,Z_1'}{Z_2 + Z_1' + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\left(\frac{Z_1\,h_{ie}}{Z_1 + h_{ie}}\right)}{Z_2 + \frac{Z_1\,h_{ie}}{Z_1 + h_{ie}} + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_1\,Z_2 h_{ie}}{Z_2(Z_1 + h_{ie}) + Z_1\,h_{ie} + Z_3(Z_1 + h_{ie})} = -1$$

$$\frac{Z_1\,Z_2 h_{fe}}{Z_2(Z_1 + h_{ie}) + Z_1\,h_{ie} + Z_3(Z_1 + h_{ie})} = -1$$

$$Z_1\,Z_2 h_{fe} = -Z_2(Z_1 + h_{ie}) - Z_1\,h_{ie} - Z_3(Z_1 + h_{ie})$$

$$Z_1\,Z_2 h_{fe} + Z_2(Z_1 + h_{ie}) + Z_1\,h_{ie} + Z_3(Z_1 + h_{ie}) = 0$$

$$Z_1\,Z_2 h_{fe} + Z_1 Z_2 + Z_2 h_{ie} + Z_1\,h_{ie} + Z_1 Z_3 + Z_3 h_{ie} = 0$$

$$Z_1\,Z_2\left(1 + h_{fe}\right) + (Z_1 + Z_2 + Z_3)h_{ie} + Z_1 Z_3 = 0 \qquad (5)$$

This is the general equation for the oscillator.

## Analysis of Hartley Oscillator

Suppose in Hartley oscillator, the resistances of inductors are negligibly small and $M$ be the mutual inductance between the inductors. Now we have

$$Z_1 = j\omega L_1 + j\omega M$$

$$Z_2 = j\omega L_2 + j\omega M$$

$$Z_3 = \frac{1}{j\omega C}$$

Substituting these values in eq. (5), we get

$$(j\omega L_1 + j\omega M)(j\omega L_2 + j\omega M)\left(1 + h_{fe}\right) + \left(j\omega L_1 + j\omega M + j\omega L_2 + j\omega M + \frac{1}{j\omega C}\right)h_{ie} + (j\omega L_1 + j\omega M)\frac{1}{j\omega C} = 0$$

$$j\omega j\omega(L_1 + M)(L_2 + M)\left(1 + h_{fe}\right) + j\omega\left(L_1 + M + L_2 + M - \frac{1}{\omega^2 C}\right)h_{ie} + j\omega(L_1 + M)\frac{1}{j\omega C} = 0$$

$$-\omega^2(L_1 + M)(L_2 + M)\left(1 + h_{fe}\right) + j\omega h_{ie}\left(L_1 + M + L_2 + M - \frac{1}{\omega^2 C}\right) + \frac{(L_1 + M)}{C} = 0 \qquad (6)$$

The frequency of oscillation can be obtained by equating the imaginary part of eq. (6) to zero. i.e.,

$$\omega h_{ie}\left(L_1 + M + L_2 + M - \frac{1}{\omega^2 C}\right) = 0$$

$$L_1 + L_2 + 2M - \frac{1}{\omega^2 C} = 0$$

$$\omega^2 = \frac{1}{(L_1 + L_2 + 2M)C}$$

$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2 + 2M)C}}$$

The condition for the maintenance of the oscillations can be obtained by equating the real part of eq.. (6) to zero. Thus

$$-\omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) + \frac{(L_1 + M)}{C} = 0$$

$$\omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) = \frac{(L_1 + M)}{C}$$

$$(1 + h_{fe}) = \frac{1}{(L_2 + M)\omega^2 C}$$

$$h_{fe} = \frac{(L_1 + L_2 + 2M)C}{(L_2 + M)C} - 1$$

$$h_{fe} = \frac{(L_1 + M)}{(L_2 + M)} \qquad (8)$$

Equation (8) gives the condition for the maintenance of the oscillations.

**Problem 1:** Find the operating frequency of a transistor Hartley oscillator if $L_1 = 100\ \mu H$. $L_2 = 1\ mH$, mutual inductance between the coils $M = 20\ \mu H$ and $C = 20\ pF$.

**Soln:** In the case of the Hartley oscillator, the frequency of oscillation is

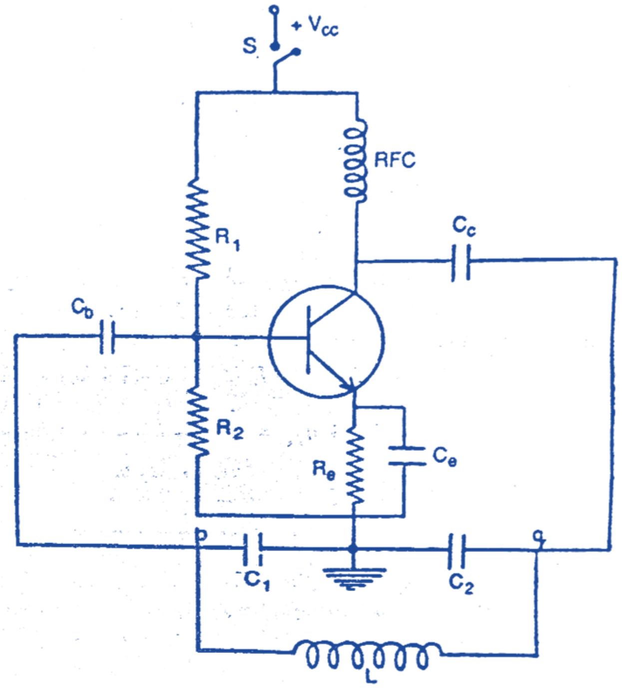
$$f = \frac{1}{2\pi\sqrt{(L_1 + L_2 + 2M)C}}$$

$$f = \frac{1}{2 \times 3.1416 \times \sqrt{(100 \times 10^{-6} + 1 \times 10^{-3} + 2 \times 20 \times 10^{-6}) \times 20 \times 10^{-12}}}$$

$$= 1052\ k\ Hz$$

## Colpitts's Oscillator

The circuit of the Colpitts oscillator is the same as that of the Hartley oscillator except that the emitter tap is connected between the capacitances $C_1$ and $C_2$. In this circuit, the parallel combination of $R_e$ and $C_e$ in conjunction with $R_1$ and $R_2$ combination provides stabilized self-bias. The frequency determining network is made up of inductance $L$ and capacitors $C_1$ and $C_2$. The function of $C$ is to block d.c. and provides an a.c. path from the collector to the tank circuit. RFC (Radio frequency choke) provides the necessary d.c. load resistance for the collector and also prevents a.c. signal from entering the d.c. supply $V_{cc}$. The condenser $C_b$ conveys feedback from collector to base circuit.




### Circuit operation

When the switch $S$ is closed, the capacitors $C_1$ and $C_2$ are charged. These capacitors are discharged through the inductance $L$ and thereby set up damped harmonic oscillations in the tank circuit. The oscillations across $C_1$ are applied to the base-emitter junction and appears in the amplified form in the collector circuit and supply losses to the tank circuit. The amount of feedback depends upon the relative capacitance values of $C_1$ and $C_2$. Higher is the value of $C_1$, smaller is the feedback.

Now we shall show that the energy supplied to the tank circuit is in phase with the generated oscillations. Hence the capacitors $C_1$ and $C_2$ act as an alternating voltage divider. So points p and q are 180° out of phase. Further, we know that a CE transistor produces a phase change of 180° between the input voltage and output voltage. Thus, a total phase change of 360° occurs. In this way, continuous undamped oscillations are produced.

### General theory of Colpitts's Oscillator

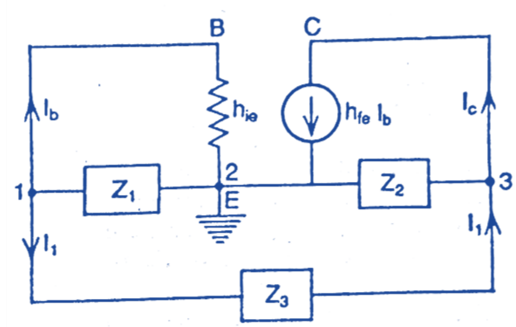
A complete theory of Colpitts's oscillator using a bipolar transistor is complicated and hence we make the following assumptions:

a) The feedback source of e.m.f., $h_{re}V_o$, is omitted because $h_{re}$ (reverse voltage ratio) of the transistor is negligible.

b) The output admittance $h_{oe}$ of the transistor is very small i.e., the output resistance $\frac{1}{h_{oe}}$ of the transistor in parallel with the inductance $L_2$ is very large. Hence $\frac{1}{h_{oe}}$ is omitted.

c) The inductive and capacitive reactances are represented by $Z_1$, $Z_2$ and $Z_3$.

d) The input terminals are taken as 1 and 2 while the output terminals are taken as 2 and 3.



The load impedance $Z_1$ and input impedance $h_{ie}$ are in parallel and hence the equivalent impedance $Z_1{}'$ is given by

$$Z_1' = \frac{Z_1\, h_{ie}}{Z_1 + h_{ie}} \qquad (1)$$

Now the load impedance $Z_L$ between output terminals 2 and 3 is equivalent to the equivalent impedance of $Z_2$ in parallel with the series combination of $Z_1{}'$ and $Z_3$. Hence

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{Z_1' + Z_3}$$

$$Z_L = \frac{Z_2\,(Z_1' + Z_3)}{Z_2 + Z_1' + Z_3} \qquad (2)$$

The voltage gain without feedback is given by

$$A_{ve} = -\frac{h_{fe}\, Z_L}{h_{ie}} = \frac{\left(\frac{\partial I_C}{\partial I_B}\right)_{V_{CE}} Z_L}{\left(\frac{\partial V_{BE}}{\partial I_B}\right)_{V_{CE}}} = \frac{\partial I_C\, Z_L}{\partial V_{BE}} = \frac{output\ voltage}{input\ voltage} \qquad (3)$$

In order to obtain the feedback fraction $\beta$, we consider the output voltage between terminals 2 and 3. The output voltage is given by

$$V_0 = I_1(Z_1' + Z_3)$$

The voltage feedback to the input terminals 1 and 2 is given by

$$V_{fb} = I_1 Z_1'$$

$$\therefore \quad \beta = \frac{V_{fb}}{V_0} = \frac{Z_1'}{Z_1' + Z_3} \qquad (4)$$

Applying the condition $A_{ve}\,\beta = 1$ for oscillation, we get

$$-\frac{h_{fe}\, Z_L}{h_{ie}} \times \frac{Z_1'}{Z_1' + Z_3} = 1$$

Substituting the value of $Z_L$, we get

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\,(Z_1' + Z_3)}{Z_2 + Z_1' + Z_3} \times \frac{Z_1'}{Z_1' + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\, Z_1'}{Z_2 + Z_1' + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_2\left(\frac{Z_1\, h_{ie}}{Z_1 + h_{ie}}\right)}{Z_2 + \frac{Z_1\, h_{ie}}{Z_1 + h_{ie}} + Z_3} = -1$$

$$\frac{h_{fe}}{h_{ie}} \times \frac{Z_1 Z_2 h_{ie}}{Z_2(Z_1 + h_{ie}) + Z_1 h_{ie} + Z_3(Z_1 + h_{ie})} = -1$$

$$\frac{Z_1 Z_2 h_{fe}}{Z_2(Z_1 + h_{ie}) + Z_1 h_{ie} + Z_3(Z_1 + h_{ie})} = -1$$

$$Z_1 Z_2 h_{fe} = -Z_2(Z_1 + h_{ie}) - Z_1 h_{ie} - Z_3(Z_1 + h_{ie})$$

$$Z_1 Z_2 h_{fe} + Z_2(Z_1 + h_{ie}) + Z_1 h_{ie} + Z_3(Z_1 + h_{ie}) = 0$$

$$Z_1 Z_2 h_{fe} + Z_1 Z_2 + Z_2 h_{ie} + Z_1 h_{ie} + Z_1 Z_3 + Z_3 h_{ie} = 0$$

$$Z_1 Z_2 (1 + h_{fe}) + (Z_1 + Z_2 + Z_3) h_{ie} + Z_1 Z_3 = 0 \tag{5}$$

This is the general equation for the oscillator.

## Analysis of Colpitts's Oscillator

Suppose in Colpitts's oscillator, the resistances of inductors are negligibly small and $M$ be the mutual inductance between the inductors. Now we have

$$Z_1 = \frac{1}{j\omega C_1}$$

$$Z_2 = \frac{1}{j\omega C_2}$$

$$Z_3 = j\omega L$$

Substituting these values in eq. (5), we get

$$\frac{1}{j\omega C_1}\frac{1}{j\omega C_2}(1 + h_{fe}) + \left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + j\omega L\right) h_{ie} + \frac{1}{j\omega C_1} j\omega L = 0$$

$$-\frac{1}{\omega^2 C_1 C_2}(1 + h_{fe}) + j\omega\left(L - \frac{1}{\omega^2 C_1} - \frac{1}{\omega^2 C_2}\right) h_{ie} + \frac{L}{C_1} = 0 \tag{6}$$

The frequency of oscillation can be obtained by equating the imaginary part to zero

$$\omega\left(L - \frac{1}{\omega^2 C_1} - \frac{1}{\omega^2 C_2}\right) h_{ie} = 0$$

$$\frac{1}{\omega^2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = L$$

$$\frac{1}{\omega^2}\left(\frac{C_1 + C_2}{C_1 C_2}\right) = L$$

$$\omega^2 = \frac{C_1 + C_2}{L C_1 C_2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{C_1 + C_2}{LC_1C_2}} \tag{7}$$

Equation (7) gives the frequency of oscillations. To obtain the condition of maintenance of oscillations, we compare the real parts on both sides of eq. (6). Hence

$$-\frac{1}{\omega^2 C_1 C_2}\left(1 + h_{fe}\right) + \frac{L}{C_1} = 0$$

$$1 + h_{fe} = \frac{L}{C_1}\omega^2 C_1 C_2$$

$$h_{fe} = \omega^2 C_2 L - 1$$

$$h_{fe} = \frac{C_1 + C_2}{LC_1C_2} C_2 L - 1$$

$$h_{fe} = \frac{C_1 + C_2}{C_1} - 1$$

$$h_{fe} = \frac{C_2}{C_1} \tag{8}$$

Equation (8) gives the condition of maintenance of oscillations.

**Problem 1:** Find the operating frequency of a transistor Colpitts's oscillator if $C_1 = 0.001\ \mu F$, $C_2 = 0.01\ \mu F$ and $L = 15\ \mu H$.

**Solution:**

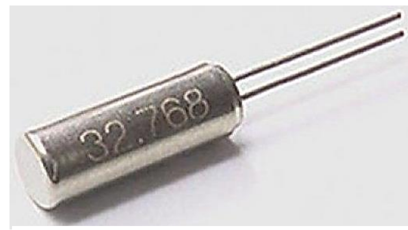
For Colpitts's oscillator, the frequency of oscillation is

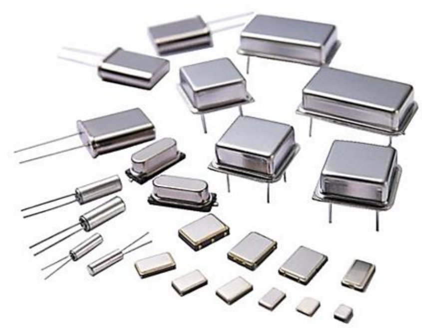
$$f = \frac{1}{2\pi}\sqrt{\frac{C_1 + C_2}{LC_1C_2}}$$

$$f = \frac{1}{2 \times 3.1416}\sqrt{\frac{0.001 \times 10^{-6} + 0.01 \times 10^{-6}}{15 \times 10^{-6} \times 0.001 \times 10^{-6} \times 0.01 \times 10^{-6}}}\ \ Hz$$

$$f = 1361 \times 10^3\ Hz = 1.361\ MHz$$

## Crystal Oscillator



We have seen that the frequency of LC oscillators depends upon the values of tank circuit parameters. These values change with time, temperature changes, etc. Hence the frequency of oscillations does not remain constant at the desired value. For excellent stability of oscillation. piezo-electric quartz crystal is used in place of the tuned circuit in the oscillator. Such an oscillator is called a crystal oscillator.

*The word piezoelectricity means electricity resulting from pressure and latent heat. It is derived from the Greek word piezein, which means to squeeze or press, and ēlektron, which means amber, an ancient source of electric charge. For example, a 1 cm$^3$ cube of quartz with 2 kN of the correctly applied force can produce a voltage of 12500 V.*

The natural shape of a quartz crystal is hexagonal. There are three axes: the z-axis is called the optic axis, the x-axis is called the electric axis and the y-axis is called the mechanical axis. The quartz crystal exhibits the property that when mechanical stress is applied across the faces of the crystal. a potential difference is developed across the opposite faces of the crystal. Conversely, if a voltage is applied across one face, mechanical stress is produced along with the other faces. This effect is called the piezo-electric effect. Thus when a piezoelectric crystal is subjected to a proper alternating potential, it vibrates mechanically. The amplitude of mechanical oscillations becomes maximum when the frequency of applied alternating voltage is equal to the natural frequency of the crystal.

The equivalent electrical circuit of a vibrating crystal can be represented by a series $LCR$ circuit shunted by $C_M$ as shown in fig. The inductor $L$ and capacitor $C$ represent electrical equivalents of crystal mass and mechanical compliance respectively. The resistance $R$ is the electrical equivalent of the crystal structure's internal friction. The shunt capacitance $C_M$ represents the capacitance due to mechanical mounting (between electrodes which are usually electroplated in position) when the crystal is not vibrating.

For a crystal with dimensions $30\,mm \times 4\,mm \times 1.5\,mm$ and at a frequency of $90\,KHz$, $L = 137H$, $C = 0.0235\,pF$, $R = 15\,K\Omega$ and $C_M = 3.5\,pF$. The impedance v/s frequency curve is shown in fig. (1c).

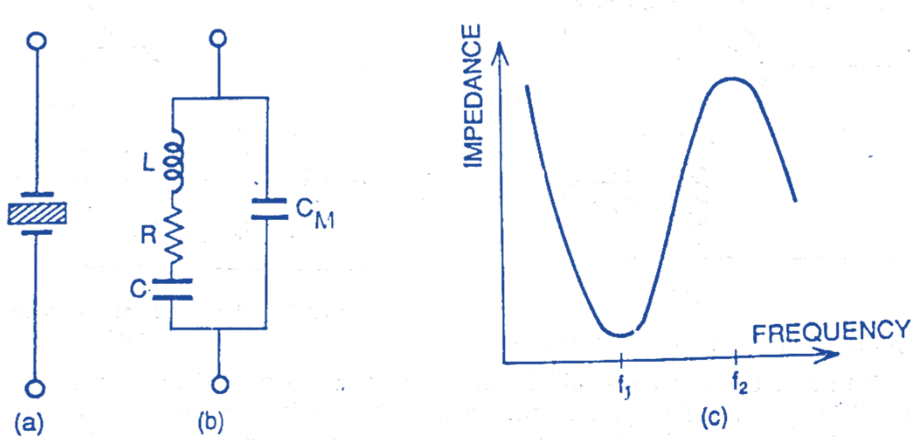



Fig. 1: Equivalent circuit of vibrating crystal

It is obvious from the figure that there exists one resonant condition when the reactances of the series RLC leg are equal and opposite. At this condition, the series-resonant impedance is very low (equal to R). Thus at series-resonance

$$|\omega_s L| = \left|\frac{1}{\omega_s C}\right|$$

$$\omega_s{}^2 = \frac{1}{LC}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

The other resonant condition occurs at a higher frequency when the reactance of the series resonant leg equals the reactance of the capacitor $C_M$. This is a parallel resonance or antiresonance condition of the crystal. At this frequency, the crystal offers a very high impedance to the external circuit. In this case,

$$\left|\omega_p L - \frac{1}{\omega_p C}\right| = \left|\frac{1}{\omega_p C_M}\right|$$

$$\omega_p L = \frac{1}{\omega_p C} + \frac{1}{\omega_p C_M}$$

$$\omega_p{}^2 = \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C_M}\right)$$

$$f_p = \frac{1}{2\pi}\sqrt{\frac{C + C_M}{LCC_M}}$$

Fig. (2) shows the crystal-controlled oscillator using a crystal in series feedback path. To excite the crystal for operation in the series-resonant mode it must be connected as a series element in the feedback path. The reason is that at series resonant frequency of the crystal its impedance is the smallest and the amount of positive feedback is the largest. In the circuit, $R_1$, $R_2$ and $R_e$ provide a voltage divider stabilized d.c. bias circuit. Capacitor $C_e$ provides a.c. bypass of emitter resistor. The capacitor C blocks any between collector and base.

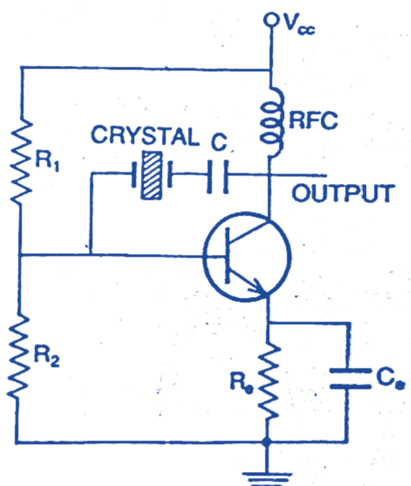



Fig. 2: Crystal controlled oscillator (series feedback)

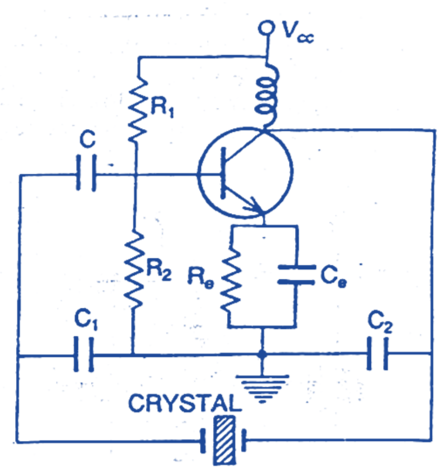



Fig. 3: Crystal controlled oscillator (parallel resonant mode)

Fig. (3) shows the crystal-controlled oscillator operating in parallel resonant mode. We know that the parallel impedance of a crystal is maximum and hence it is connected in the shunt. At the parallel resonant operating frequency, a crystal appears as an inductive reactance of the largest value. If we compare this circuit with Colpitts's oscillator circuit, the two circuits are identical except that the crystal takes the place of the inductor. It is important to note that the frequency of the oscillator is determined by the crystal parameters. Since the resistance R of the crystal is very small, its Q is very high. Hence the frequency stability of the oscillator is very high.

**Problem 1:** The a.c. equivalent circuit of a crystal has the values: $L = 1\,H$, $C = 0.01\,pF$. $R = 1000\,\Omega$, and $C_M = 20\,pF$, Calculate $f_s$ and $f_p$ of the crystal.

## References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