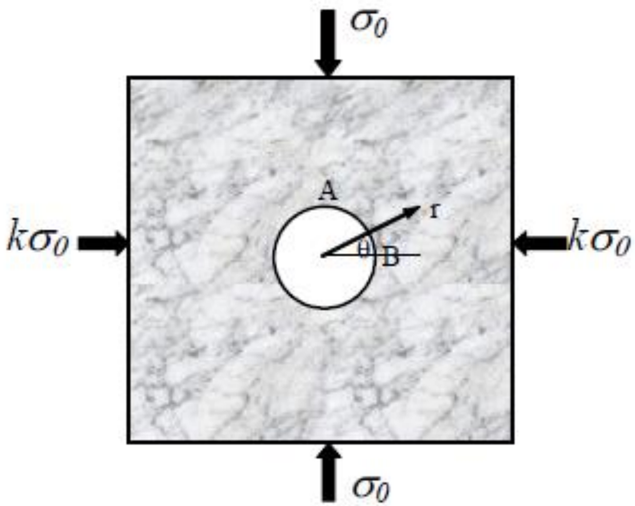
Analytical solutions of underground excavations

Circular excavation :

Biaxial stress is examined, as shown in Fig.



The biaxial stresses in the far-field of the excavation are assumed as σ_0 and $k\sigma_0$. The analytical solutions for stress and displacement distributions around the circular opening can be expressed as follows (Brady and Brown 1985):

Total radial, tangential, and shear stresses after excavation:

$$\begin{cases} \sigma_r = \dfrac{\sigma_0}{2}\left[(1+k)\left(1-\dfrac{a^2}{r^2}\right)-(1-k)\left(1-\dfrac{4a^2}{r^2}+\dfrac{3a^4}{r^4}\right)\cos 2\theta\right] \\ \sigma_\theta = \dfrac{\sigma_0}{2}\left[(1+k)\left(1+\dfrac{a^2}{r^2}\right)+(1-k)\left(1+\dfrac{3a^4}{r^4}\right)\cos 2\theta\right] \\ \tau_{r\theta} = \dfrac{\sigma_0}{2}\left[(1-k)\left(1+\dfrac{2a^2}{r^2}-\dfrac{3a^4}{r^4}\right)\sin 2\theta\right] \end{cases} \tag{11.1}$$

where a is the excavation radius; r is the distance from the center of the excavation; σ_r, σ_θ are the radial and tangential stress components, respectively; $\tau_{r\theta}$ is the shear stress component; θ is the angle as shown in Fig.

σ_0 and $k\sigma_0$ are the vertical and horizontal stresses

Radial and tangential displacements induced by excavation can be expressed as follows:

$$\begin{cases} u_r = -\dfrac{\sigma_0 a^2}{4Gr}\left[(1+k)-(1-k)\left(2(1-2\nu)+\dfrac{a^2}{r^2}\right)\cos 2\theta\right] \\ u_\theta = -\dfrac{\sigma_0 a^2}{4Gr}\left[(1-k)\left(2(1-2\nu)+\dfrac{a^2}{r^2}\right)\sin 2\theta\right] \end{cases} \tag{11.2}$$

where u_r, u_θ are the radial and tangential displacements, respectively; ν is the Poisson's ratio; and G is the shear modulus.

The induced stresses on the excavation wall can be obtained by putting $r = a$ in Eq. 11.1:

$$\begin{cases} \sigma_r = 0 \\ \sigma_\theta = \sigma_0\left[(1+k)+2(1-k)\cos 2\theta\right] \\ \tau_{r\theta} = 0 \end{cases} \tag{11.3}$$

At the crown (point A) and sidewall (point B) (Fig. 11.1) the excavation induced stresses are:

At point A:

$$\sigma_\theta = (3k - 1)\sigma_0 \tag{11.3}$$

At point B:

$$\sigma_\theta = (3 - k)\sigma_0 \tag{11.4}$$

For a horizontal tunnel, the vertical stress component (σ_0) is the overburden stress, which is $\sigma_0 = \gamma H$, and the constant k can be estimated by $k = \nu/(1-\nu)$. The stresses at the crown and sidewall can be rewritten as:

At point A:

$$\sigma_\theta = \left(\frac{4\nu - 1}{1-\nu}\right)\gamma H \tag{11.5}$$

At point B:

$$\sigma_\theta = \left(\frac{3 - 4\nu}{1-\nu}\right)\gamma H \tag{11.6}$$

where H is the depth of the excavation; and γ is the average specific weight of the rocks.

Elastoplastic solution of a circular excavation

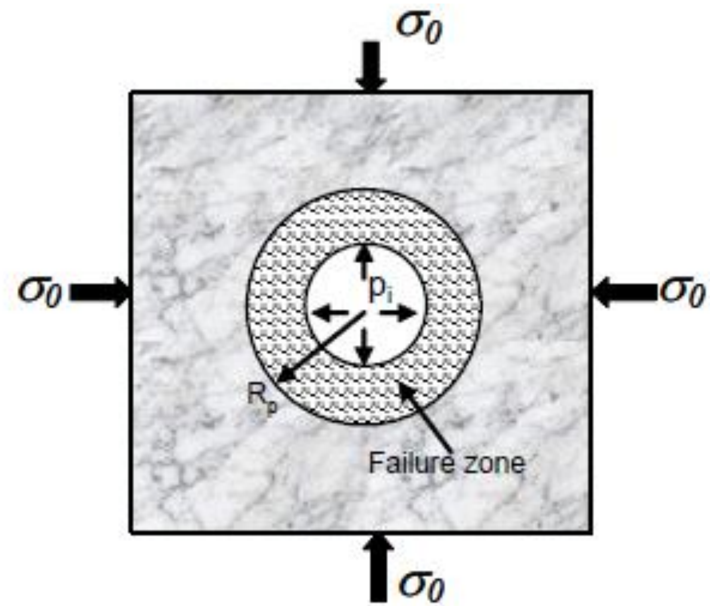


Fig. 11.2. Two-dimensional circular excavation under hydrostatic stresses and support pressure.

With a circular opening excavated in a formation subject to hydrostatic stress (σ_0) as shown in Fig. 11.2, Ladanyi (1974) gave an elastoplastic analysis on stress distributions and applied support pressure (p_i). It is assumed that the rock strength is described by the Mohr-Coulomb criterion (Brady and Brown 1985), i.e.:

$$\sigma_\theta = q\sigma_r + UCS \tag{11.7}$$

Where σ_θ and σ_r are the tangential and radial stresses, respectively; UCS is the uniaxial compressive strength of the rock; $q = (1+\sin\phi)/(1-\sin\phi)$; and ϕ is the internal friction angle of the rock.

The strength of fractured rock (failure zone) as shown in Fig. 11.2 is taken to be purely frictional, with the limiting state of stress within the fractured rock mass defined by (Brady and Brown 1985):

$$\sigma_\theta = q_f \sigma_r \tag{11.8}$$

where $q_f = (1+\sin\phi_f)/(1-\sin\phi_f)$; and ϕ_f is the internal friction angle of the fractured rock.

Applying Eq. 11.8 the stress distributions in the failure zone can be expressed as follows (Brady and Brown 1985):

$$\begin{cases} \sigma_r = p_i \left(\dfrac{r}{a}\right)^{q_f - 1} \\ \sigma_\theta = q_f p_i \left(\dfrac{r}{a}\right)^{q_f - 1} \end{cases} \tag{11.9}$$

The stress distributions in the elastic zone can be written as (Brady and Brown 1985):

$$\begin{cases} \sigma_r = \sigma_0\left(1 - \dfrac{R_p^2}{r^2}\right) + p_i \dfrac{R_p^2}{r^2} \\ \sigma_\theta = \sigma_0\left(1 + \dfrac{R_p^2}{r^2}\right) - p_i \dfrac{R_p^2}{r^2} \end{cases} \tag{11.10}$$

The radius of the fractured zone induced by excavation can be expressed as:

$$R_p = a\left[\frac{2\sigma_o - UCS}{(1+q)p_i}\right]^{1/(q_f - 1)} \tag{11.11}$$

It can be seen from Eq. 11.11 that the excavation has a smaller fractured zone with a larger support pressure and higher rock strength.

Most underground openings are non-circular, and it is difficult to obtain closed form solutions. The numerical methods, such as the finite element method, can then be used to analyze excavation-induced stress and displacement distributions and determine support pressure.