PROBLEM SOLUTION:

Normal and shear stresses and their directions

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Problem-3 with solution

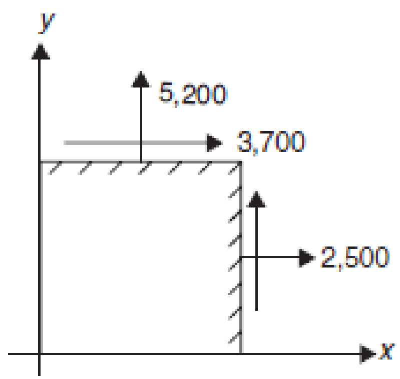
Consider a "two-dimensional" state of stress in the x–y plane characterized by:

$$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$$

where units are psi and tension is positive. Find: the magnitude and direction of the principal stresses and illustrate with a sketch.

Given: 2D stress state with tension (+)
$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$ and units in psi.

Find: σ_1, σ_3 and direction.



Solution:

From notes

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$$

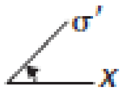
$$= \frac{2{,}500+5{,}200}{2} \pm \left[\left(\frac{2{,}500-5{,}200}{2}\right)^2 + (3{,}700)^2\right]^{1/2}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 3{,}850 \pm [3{,}939]$$

$$\underline{\underline{\sigma_1 = 7{,}789 \text{ psi}}}$$

$$\underline{\underline{\sigma_3 = -89 \text{ psi}}}$$

$$\underline{\underline{\alpha = +55^\circ}}$$



$$\tan 2\alpha = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}$$

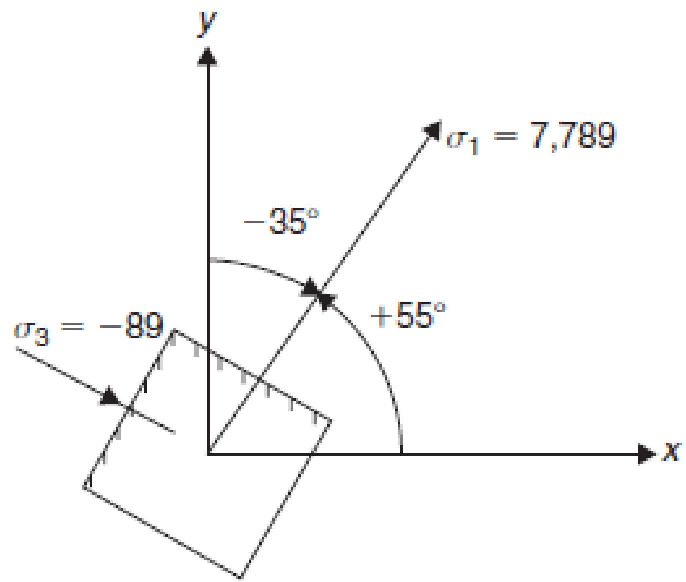
$$= \frac{3{,}700}{\frac{1}{2}(2{,}500 - 5{,}000)}$$

$$\tan 2\alpha^* = -2.7407$$

$$2\alpha^* = -70$$

$$\alpha^* = -35^\circ$$

$$\text{or} \quad \alpha^* = +55^\circ$$



find: the magnitude and direction of the maximum shear stress and illustrate with a sketch.

$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}200$ psi

(tension+)

Find: τ_{max}.

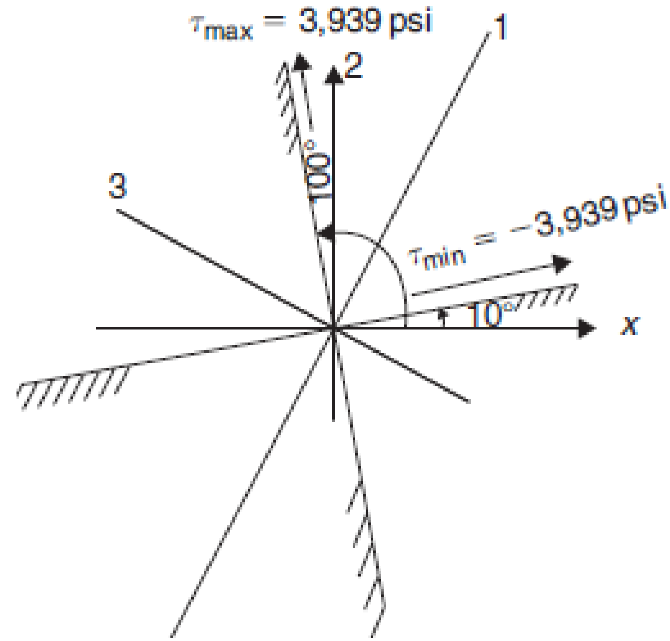
Solution:

From notes

$$\tau_{max} = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \left[\left(\frac{2{,}500 - 5{,}200}{2}\right)^2 + (3{,}200)^2\right]^{1/2}$$

$$\underline{\underline{\tau_{max} = 3{,}939 \text{ psi}}}$$



$$\tan 2\alpha^{**} = \frac{-\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}{\tau_{xy}}$$

$$= \frac{-\left(\frac{2{,}500-5{,}200}{2}\right)}{3{,}700}$$

$$\tan 2\alpha^{**} = 0.36486$$

$$2\alpha^{**} = 20$$

$$\alpha^{**} = 10^\circ$$

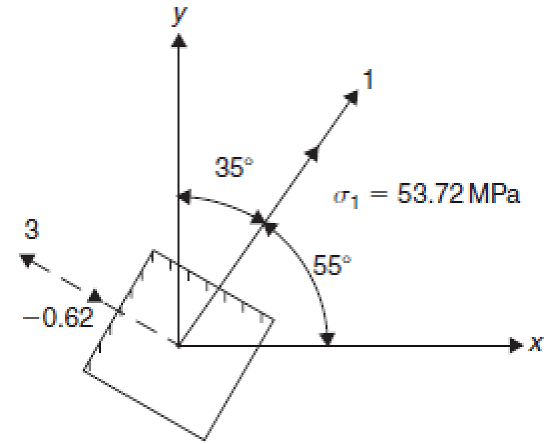
$$\text{or} \quad \alpha^{**} = 100^\circ$$

Problem-4 with solution

Consider a "two-dimensional" state of stress in the x–y plane characterized by:

$$\sigma_{xx} = 17.24 \quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$$

where units are MPa and tension is positive. Find: the magnitude and direction of the principal stresses and illustrate with a sketch.



Given: 2D stress state with tension (+)

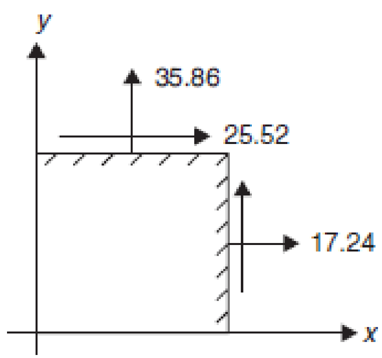
$\sigma_{xx} = 17.24$ Mpa $\quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$

Find: $\sigma_1, \sigma_3, \alpha$.

Solution:

Formulas:
$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

$$= \frac{17.74 + 35.86}{2} \pm \left[\left(\frac{17.24 - 35.86}{2}\right)^2 + (25.52)^2\right]^{1/}$$



$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = 26.55 \pm [27.17]$$

$$\therefore \left.\begin{matrix}\underline{\underline{\sigma_1 = 53.72\ \text{MPa}}}\\ \underline{\underline{\sigma_3 = -0.62\ \text{MPa}}}\end{matrix}\right\}$$

$\underline{\underline{+55^\circ \angle}}$ 3, x

σ_1, σ_3 ←

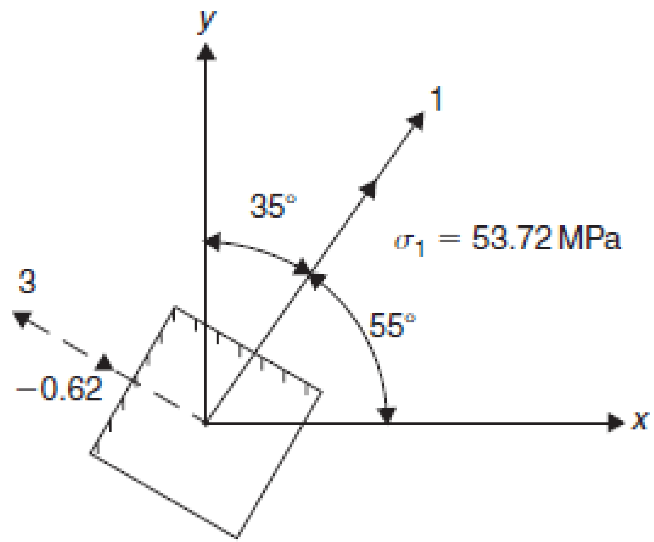
$$\tan 2\alpha = \frac{\tau_{xy}}{\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}$$

$$= \frac{(2)(25.52)}{17.24 - 35.86}$$

$$\tan 2\alpha = -2.741$$

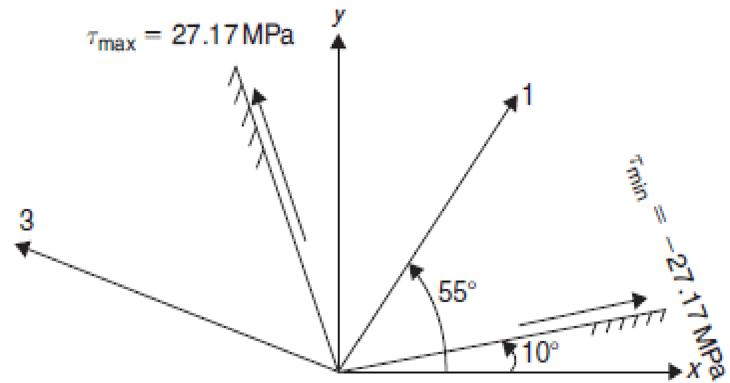
$$2\alpha = -70.0^\circ, 110^\circ$$

$$\alpha = -35^\circ, +55^\circ$$



find: the magnitude and direction of the maximum shear stress and illustrate with a sketch.

Given: Problem 13 data:
$\sigma_{xx} = 17.24$ MPa $\quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$
Find: τ_{max}, α^{**}.

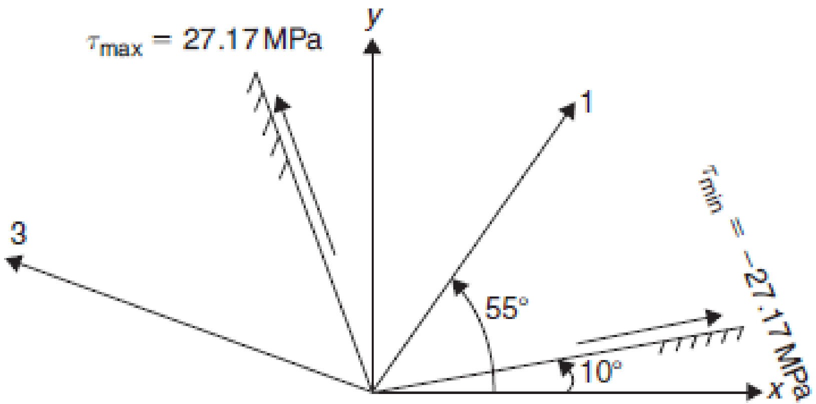


Formulas

$$\tau_{max} = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2} \right)^2 + (\tau_{xy})^2 \right]^{1/2}$$

$$= \left[\left(\frac{17.24 - 35.86}{2} \right)^2 + (25.52)^2 \right]^{1/2}$$

$$\underline{\underline{\tau_{max} = 27.17 \text{ MPa}}}$$



$$\tan 2\alpha^{**} = \frac{-\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}{\tau_{xy}}$$

$$= \frac{-\frac{1}{2}(17.24 - 35.86)}{25.52}$$

$$\tan 2\alpha^{**} = 0.3648$$

$$2\alpha^{**} = 20.0^\circ, 200^\circ$$

$$\underline{\underline{\alpha^{**} = 10^\circ, 100^\circ}}$$

Problem-5 with solution

Suppose that

$$\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$$

and the z-direction shear stresses (τ_{zx}, τ_{zy}) are zero, while the z-direction normal stress (σ_{zz}) is 4,000 psi. Find: the major, intermediate, and minor principal stresses.

Given: $\sigma_{xx} = 2{,}500 \quad \sigma_{yy} = 5{,}200 \quad \tau_{xy} = 3{,}700$ psi and
$\tau_{xz} = 0 \quad \tau_{yz} = 0 \quad \sigma_{zz} = 400$ psi
Find: σ_1, σ_2, σ_3.

Solution:

Assume tension is positive. The plane that σ_{zz} acts on is shear-free and therefore is a principal plane. Thus, σ_{zz} is a principal stress. Also (from prob. 11) in the xy-plane

$$\sigma_1 = 7{,}789 \;\&\; \sigma_3 = -89 \text{ psi}$$

$$\therefore \underline{\underline{\sigma_1 = 7{,}789, \quad \sigma_2 = 4{,}000(\sigma_{zz}) \;\&\; \sigma_3 = -89 \text{ psi}}} \qquad \longleftarrow$$

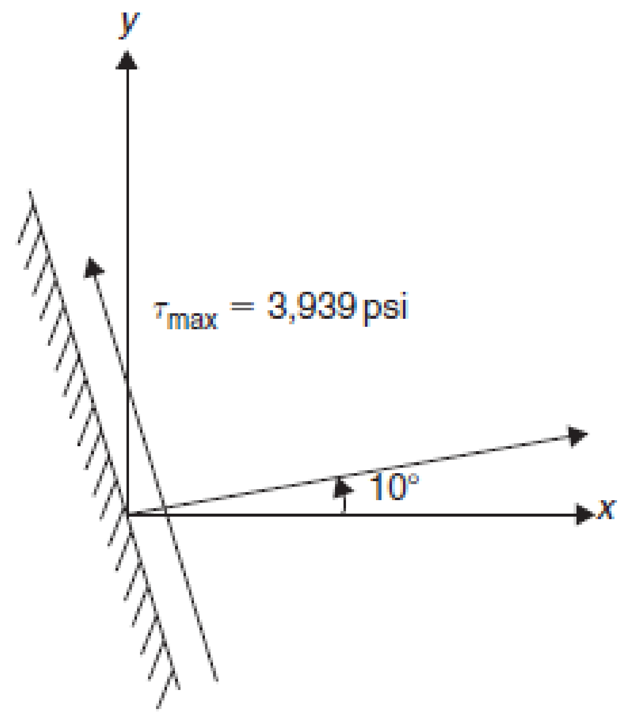
Find: The maximum shear stress and orientation of associated plane.

Solution:

Since $\tau_{xz} = \tau_{yz} = 0$, it seems

$\underline{\underline{\tau_{max} = 3{,}939 \text{ psi}}} : [= \frac{1}{2}(\sigma_1 - \sigma_3)]$

$\underline{\underline{\alpha^{**} = 10^\circ}}$ $\therefore$ must see in x-y view



Problem-6 with solution

Suppose that

$$\sigma_{xx} = 17.24 \quad \sigma_{yy} = 35.86 \quad \tau_{xy} = 25.52$$

in MPa and the z-direction shear stresses (τ_{zx}, τ_{zy}) are zero, while the z-direction normal stress (σ_{zz}) is 27.59 MPa. Find: the major, intermediate, and minor principal stresses.

Given: $\sigma_{xx} = 17.24$ MPa $\quad \sigma_{yy} = 35.86$ MPa $\quad \sigma_{zz} = 27.59$ MPa $\quad \tau_{xy} = 25.52$
$\tau_{yz} = 0 \quad \tau_{zx} = 0$
Find: $\sigma_1, \sigma_2, \sigma_3$.

Solution:

Assume tension is positive. The plane that σ_{zz} acts on is a shear-free plane ($\tau_{yz} = \tau_{zx} = 0$) and therefore is a principal plane. Thus, σ_{zz} is a principal stress.

Formulas: $$\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2\right]^{1/2}$$

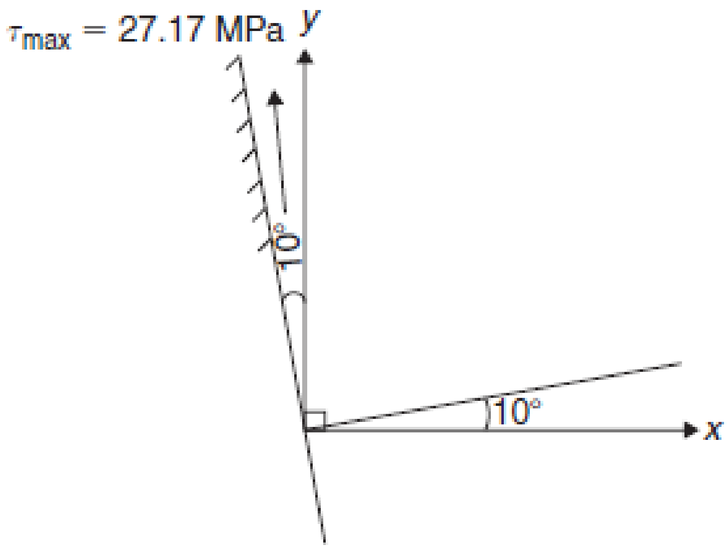
$$\left.\begin{matrix}\sigma_1 = 53.72 \text{ MPa} \\ \sigma_3 = -0.62 \text{ MPa}\end{matrix}\right\} \text{in } x\text{-}y \text{ plane}$$

$$\sigma_2 = \sigma_{zz} = 27.59 \text{ MPa}$$

$$\therefore \underline{\underline{\sigma_1 = 53.72, \sigma_2 = 27.59, \sigma_3 = -0.62 \text{ MPa}}}$$

Find: Maximum shear stress and orientation

Solution:



Formulas

$$\tau_{max} = \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2} \right)^2 + (\tau_{xy})^2 \right]^{1/2}$$

$$\tan 2\alpha^{**} = \frac{-\frac{1}{2}(\sigma_{xx} - \sigma_{yy})}{\tau_{xy}}$$

Because $\tau_{yz} = \tau_{zx} = 0$

and $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3)$

$\underline{\tau_{max} = 27.17 \text{ MPa}}$

$\underline{\alpha^{**} = 10^\circ}$ ←