Lecture-4

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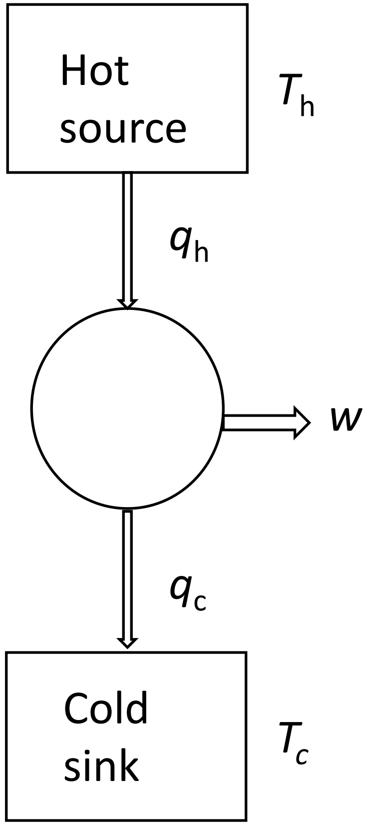
Heat Engines

- A **heat engine** converts some of the random molecular energy of heat flow into macroscopic mechanical energy (work).
- The working substance is heated in a cylinder, and its expansion moves a piston, thereby doing mechanical work.
- Suppose an energy q_h = 20 kJ is supplied to the engine and q_c = -15 kJ is lost from the engine and discarded into the cold reservoir. The work done by the engine = $q_h + q_c$ = 20 kJ + (−15 kJ) = 5 kJ. The efficiency of the heat engine= 5 kJ/20 kJ = 0.25 or 25%.

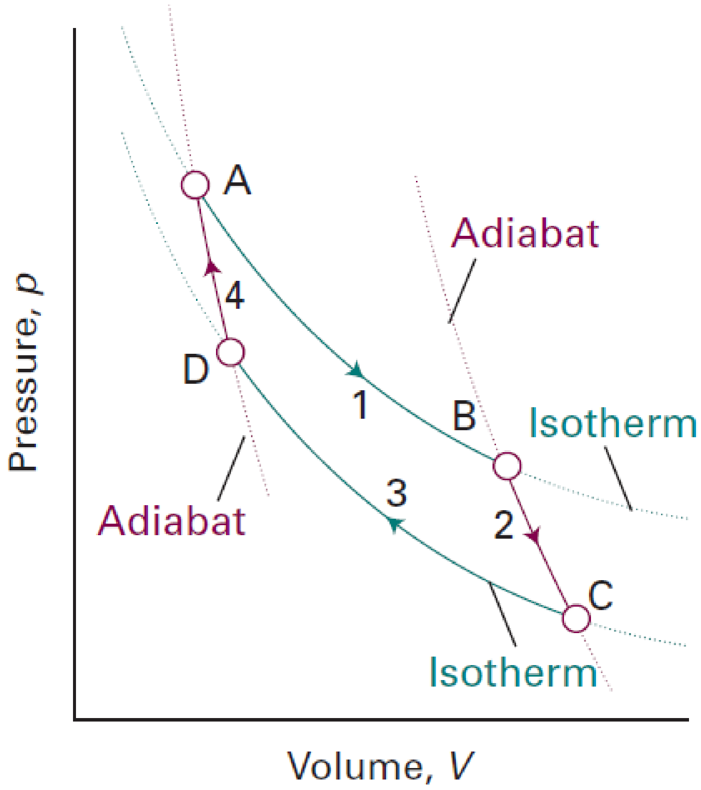


Carnot Cycle

A Carnot cycle, which is named after the French engineer Sadi Carnot, consists of four reversible stages :

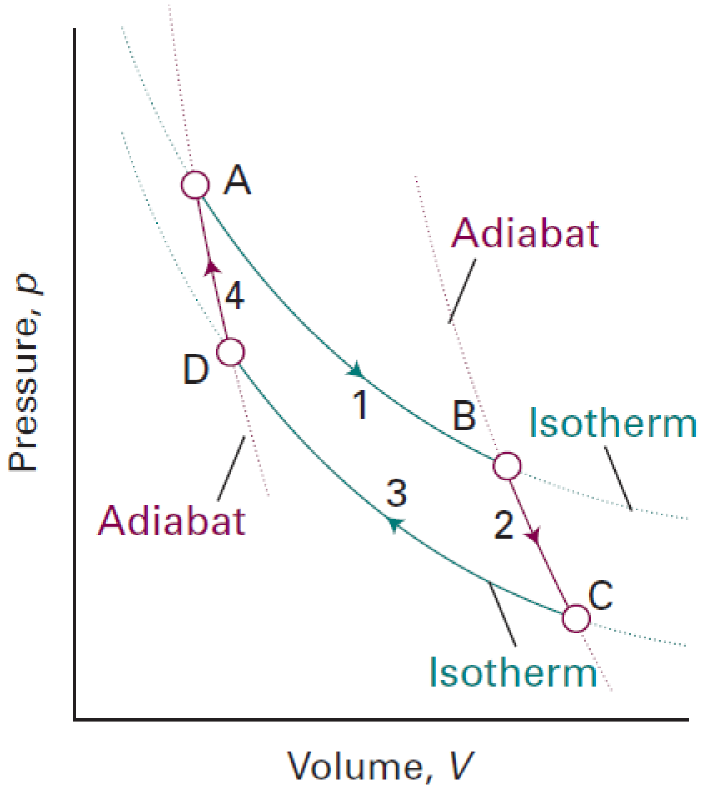
1. Reversible isothermal expansion from A to B at T_h; the entropy change is q_h/T_h, where q_h is the energy supplied to the system as heat from the hot source.

2. Reversible adiabatic expansion from B to C. No energy leaves the system as heat, so the change in entropy is zero. In the course of this expansion, the temperature falls from T_h to T_c, the temperature of the cold sink.



3. Reversible isothermal compression from C to D at T_c. Energy is released as heat to the cold sink; the change in entropy of the system is q_c / T_c; in this expression q_c is negative.

4. Reversible adiabatic compression from D to A. No energy enters the system as heat, so the change in entropy is zero. The temperature rises from T_c to T_h.



For isothermal stages (step 1 and step 3)

$$q_h = nRT_h \ln \frac{V_B}{V_A} \qquad q_c = nRT_c \ln \frac{V_D}{V_C}$$

For adiabatic steps (step 2 and step 4) VT^c = constant

$$V_A T_h^c = V_D T_c^c \qquad V_C T_c^c = V_B T_h^c$$

Multiplication of the first of these expression with the second gives

$$V_A V_C T_h^c T_c^c = V_D V_B T_h^c T_c^c$$

Which simplifies to

$$\frac{V_A}{V_B} = \frac{V_D}{V_C}$$

With these relations established, we can write

$$q_c = nRT_c \ln \frac{V_D}{V_C} = nRT_c \ln \frac{V_A}{V_B} = -nRT_c \ln \frac{V_B}{V_A}$$

and therefore,

$$\frac{q_h}{q_c} = \frac{nRT_h \ln(V_B/V_A)}{-nRT_c \ln(V_B/V_A)} = -\frac{T_h}{T_c}$$

or, $\dfrac{q_h}{q_c} = -\dfrac{T_h}{T_c}$ or, $\dfrac{q_h}{T_h} = -\dfrac{q_c}{T_c}$

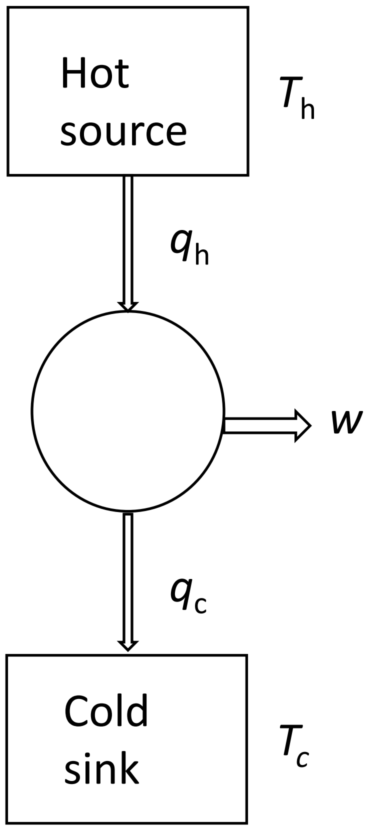
Thus, for Carnot cycle

$$\oint dS = \frac{q_h}{T_h} + \frac{q_c}{T_c} = 0$$

Efficiency (η) of Carnot heat engine

The efficiency, η, of heat engine of a heat engine is defined as-

$$\eta = \frac{\text{work performed}}{\text{heat absorbed from hot source}} = \frac{|w|}{|q_h|}$$



The work performed by the engine is the difference between the energy supplied as heat by the hot reservoir and return to the cold reservoir. Therefore,

$$\eta = \frac{|q_h| - |q_c|}{|q_h|} = 1 - \frac{|q_c|}{|q_h|}$$

As $q_h/q_c = -T_h/T_c$ and noting that the modulus sign removes the negative sign one obtain

$$\eta = 1 - \frac{T_c}{T_h}$$

Important notes on efficiency (η) of a Carnot cycle

- Maximum work can be obtained from a reversible Carnot cycle.
- Efficiency of the reversible Carnot cycle depends only on the temperature of the source and the sink.
- The Carnot cycle efficiency can be used to make an estimate of the maximum conversion of heat into work that can be expected for a real engine.
- If $T_h = \infty$ or $T_c = 0K$, then $\eta = 1$. So complete conversion of heat into work can be effected only if the hot source temperature is ∞ or the sink can be placed at 0K, which can not be achieved. Thus we can not have any heat engine with unit efficiency.

Efficiency of Steam engine

- Most of our electric power is produced by steam engines (more accurately, steam turbines) that drive conducting wires through magnetic fields, thereby generating electric currents. A modern steam power plant might have the boiler at 550°C and the condenser at 40°C.
- If it operates on a Carnot cycle, then

$$\eta = 1 - (313\text{ K})/(823\text{ K}) = 62\%.$$

- The actual cycle of a steam engine is not a Carnot cycle because of irreversibility and because heat is transferred at temperatures between T_h and T_c. These factors make the actual efficiency less than 62%.
- The efficiency of a modern steam power plant is typically about 40%. (For comparison, James Watt's steam engines of the late 1700s had an efficiency of roughly 15%.)

All reversible engines have the same efficiency

- Initially, suppose that engine A is more efficient than engine B, and that we choose a setting of the controls that causes engine B to acquire energy as heat q_c from the cold reservoir and to release a certain quantity of energy as heat into the hot reservoir.
- However, because engine A is more efficient than engine B, not all the work that A produces is needed for this process.
- The net result is that the cold reservoir is unchanged, the hot reservoir has lost a certain amount of energy and the lost heat energy has been converted directly into work.
- This outcome is contrary to the Kelvin statement of the Second Law, the initial assumption must be false. In other words, all reversible engines have the same efficiency.